Positive Solution for m-Point Boundary Value Problems

Wing-Sum Cheung[1]
Department of Mathematics, The University of Hong Kong
Pokfulam, Hong Kong
Email: wscheung@hkucc.hku.hk
Tel: (852) 2859 1996
Fax: (852) 2559 2225
and
Jingli Ren[2]
Department of Mathematics, Zhengzhou University
Zhengzhou, Henan 450052, P.R. China
Email: ren_jingli@163.com

Abstract— In this paper, we investigate an m-point boundary value problem with sign changing nonlinearity. The existence of an interval of parameters which ensures the problem has at least one positive solution is determined by constructing available operator and combining the method of lower solution with the method of topology degree. Moreover, the associated Green's function for the above problem is also given.

Keywords— m-point boundary value problems, Green's function, operator, cone.

[1]Correspondence author. Research is partically supported by the Research Grants Council of the Hong Kong SAR, China (Project No. HKU7040/03P)

[2]Research is supported by the National Natural Science Foundation of China (19871005).

1. INTRODUCTION

In recent years, the existence of positive solutions for multi-point boundary value problems attracts certain authors' attention, see [1-4] and reference therein. The results have been obtained mainly by the fixed-point theorem in cones, such as Kransnosel'skii fixed-point theorem[5], Leggett-Williams' theorem[6], Avery and Henderson's theorem[7], and so on. In order to applied the concavity of solutions in the proofs, all the above works were done under the assumption that the nonlinear term is nonnegative. For example, in [3], Youssef N. Raffoul studied a three-point boundary value problems(BVP)

$$u''(t) + \lambda a(t) f(u) = 0, \quad 0 \le t \le 1, \tag{1.1}$$

$$u(0) = 0, \quad u(1) = \alpha u(\eta), \tag{1.2}$$

where $0 < \eta < 1$, $0 < \alpha < \frac{1}{\eta}$, $a \in C([0,1],[0,\infty))$, and $f \in C([0,\infty),[0,\infty))$. The author applied Kransnosel'skii fixed-point theorem and obtained conditions for the existence of positive solutions to BVP (1.1)-(1.2).

In this paper we let $m \ge 3$ be a fixed integer and consider the following m-point boundary value problem

$$u''(t) + \lambda h(t) f(t,u) = 0, \quad 0 \le t \le 1, \tag{1.3}$$

$$u(0) = 0, \quad u(1) = \sum_{i=1}^{m-2} \alpha_i u(\xi_i), \tag{1.4}$$

here $0 < \xi_1 < \xi_2 < \cdots < \xi_{m-2} < 1$, the nonlinear term f is continuous and is allowed to change sign. First we give the associated Green's function for the above problem which makes later discussions more precise. Then by constructing available operators, we combine the method of lower solution with the method of topology degree and show that BVP (1.3)-(1.4) has at least one positive solution with certain growth conditions imposed on f. In this way we removed the usual restriction $f \ge 0$.

2. PRELIMINARY

Before the statement of our main results, we give some lemmas which are needed later.

LEMMA 2.1. Suppose that $\sum\limits_{i=1}^{m-2} \alpha_i \xi_i \ne 1$, $y(t) \in C[0,1]$, then BVP

$$u'' + y(t) = 0, \quad 0 \le t \le 1, \tag{2.1}$$

$$u(0) = 0, \quad u(1) = \sum_{i=1}^{m-2} \alpha_i u(\xi_i) \tag{2.2}$$

has a unique solution

$$\begin{aligned} u(t) \;=\; & -\int_0^t (t-s) y(s) ds + \frac{t}{1 - \sum\limits_{i=1}^{m-2} \alpha_i \xi_i} \int_0^1 (1-s) y(s) ds \\ & - \frac{t}{1 - \sum\limits_{i=1}^{m-2} \alpha_i \xi_i} \sum_{i=1}^{m-2} \alpha_i \int_0^{\xi_i} (\xi_i - s) y(s) ds. \end{aligned} \tag{2.3}$$

PROOF. Integrating both sides of (2.1) on $[0, t]$, we have

$$u'(t) = -\int_0^t y(s)ds + u'(0). \tag{2.4}$$

Again integrating (2.4) from 0 to t, making use of the condition that $u(0) = 0$, we have

$$u(t) = -\int_0^t (t-s)y(s)ds + u'(0)t. \tag{2.5}$$

In particular,

$$u(1) = -\int_0^1 (1-s)y(s)ds + u'(0),$$

and

$$u(\xi_i) = -\int_0^{\xi_i} (\xi_i - s)y(s)ds + u'(0)\xi_i, \quad i = 1, 2, \cdots, m-2.$$

By (2.2), we get

$$u'(0) = \frac{1}{1 - \sum\limits_{i=1}^{m-2} \alpha_i\xi_i} \int_0^1 (1-s)y(s)ds - \frac{1}{1 - \sum\limits_{i=1}^{m-2} \alpha_i\xi_i} \sum_{i=1}^{m-2} \alpha_i \int_0^{\xi_i} (\xi_i - s)y(s)ds. \tag{2.6}$$

Then Lemma 2.1 is proved. □

LEMMA 2.2. Suppose $\sum\limits_{i=1}^{m-2} \alpha_i\xi_i \neq 1$, then the Green's function for the BVP

$$-u'' = 0, \quad 0 \le t \le 1, \tag{2.7}$$

$$u(0) = 0, \quad u(1) = \sum_{i=1}^{m-2} \alpha_i u(\xi_i), \tag{2.8}$$

is given by

$$G(t,s) = \begin{cases} \dfrac{s(1-t) - \sum\limits_{j=i}^{m-2} \alpha_j(\xi_j - t)s + \sum\limits_{j=1}^{i-1} \alpha_j\xi_j(t-s)}{1 - \sum\limits_{i=1}^{m-2} \alpha_i\xi_i}, \\ \quad \text{for } \ 0 \le t \le 1, \ \xi_{i-1} \le s \le \min\{\xi_i, t\}, \ i = 1, 2, \cdots, m-1; \\ \\ \dfrac{t[(1-s) - \sum\limits_{j=i}^{m-2} \alpha_j(\xi_j - s)]}{1 - \sum\limits_{i=1}^{m-2} \alpha_i\xi_i}, \\ \quad \text{for } \ 0 \le t \le 1, \ \max\{\xi_{i-1}, t\} \le s \le \xi_i, \ i = 1, 2, \cdots, m-1. \end{cases} \tag{2.9}$$

Here for the sake of convenience, we write $\xi_0 = 0$, $\xi_{m-1} = 1$ and $\sum\limits_{i=m_1}^{m_2} f(i) = 0$, for $m_2 < m_1$.

PROOF. If $0 \le t \le \xi_1$, the unique solution (2.3) given by Lemma 2.1 can be rewritten as

$$\begin{aligned}
u(t) &= -\int_0^t (t-s)y(s)ds \quad + \frac{t}{1-\sum\limits_{i=1}^{m-2}\alpha_i\xi_i}\int_0^1(1-s)y(s)ds - \frac{t}{1-\sum\limits_{i=1}^{m-2}\alpha_i\xi_i}\sum_{j=1}^{m-2}\alpha_j\int_0^{\xi_j}(\xi_j-s)y(s)ds \\
&= -\int_0^t (t-s)y(s)ds \quad + \frac{t}{1-\sum\limits_{i=1}^{m-2}\alpha_i\xi_i}\int_0^t(1-s)y(s)ds - \frac{t}{1-\sum\limits_{i=1}^{m-2}\alpha_i\xi_i}\sum_{j=1}^{m-2}\alpha_j\int_0^{t}(\xi_j-s)y(s)ds \\
&\quad + \frac{t}{1-\sum\limits_{i=1}^{m-2}\alpha_i\xi_i}\int_t^{\xi_1}(1-s)y(s)ds - \frac{t}{1-\sum\limits_{i=1}^{m-2}\alpha_i\xi_i}\alpha_j\int_t^{\xi_1}(\xi_j-s)y(s)ds \\
&\quad + \frac{t}{1-\sum\limits_{i=1}^{m-2}\alpha_i\xi_i}\int_{\xi_1}^{\xi_2}(1-s)y(s)ds - \frac{t}{1-\sum\limits_{i=1}^{m-2}\alpha_i\xi_i}\sum_{i=2}^{m-2}\alpha_i\int_{\xi_1}^{\xi_2}(\xi_i-s)y(s)ds \\
&\quad + \cdots\cdots \\
&\quad + \frac{t}{1-\sum\limits_{i=1}^{m-2}\alpha_i\xi_i}\int_{\xi_{m-3}}^{\xi_{m-2}}(1-s)y(s)ds - \frac{t}{1-\sum\limits_{i=1}^{m-2}\alpha_i\xi_i}\sum_{i=m-2}^{m-2}\alpha_i\int_{\xi_{m-3}}^{\xi_{m-2}}(\xi_i-s)y(s)ds \\
&\quad + \frac{t}{1-\sum\limits_{i=1}^{m-2}\alpha_i\xi_i}\int_{\xi_{m-2}}^{1}(1-s)y(s)ds
\end{aligned}$$

$$\begin{aligned}
&= \int_0^t \frac{s(1-t)-\sum\limits_{j=1}^{m-2}\alpha_j(\xi_j-t)s}{1-\sum\limits_{i=1}^{m-2}\alpha_i\xi_i}y(s)ds + \int_t^{\xi_1}\frac{t[(1-s)-\sum\limits_{j=1}^{m-2}\alpha_j(\xi_j-s)]}{1-\sum\limits_{i=1}^{m-2}\alpha_i\xi_i}y(s)ds \\
&\quad + \sum_{i=2}^{m-2}\int_{\xi_{i-1}}^{\xi_i}\frac{t[(1-s)-\sum\limits_{j=i}^{m-2}\alpha_j(\xi_j-s)]}{1-\sum\limits_{i=1}^{m-2}\alpha_i\xi_i}y(s)ds + \int_{\xi_{m-2}}^1\frac{t(1-s)}{1-\sum\limits_{i=1}^{m-2}\alpha_i\xi_i}y(s)ds.
\end{aligned}$$

Similarly, if $\xi_{r-1} \le t \le \xi_r$, $\ 2 \le r \le m-2$, the unique solution (2.3) can be rewritten as

$$\begin{aligned}
u(t) &= \int_0^{\xi_1}\frac{s(1-t)-\sum\limits_{j=1}^{m-2}\alpha_j(\xi_j-t)s}{1-\sum\limits_{i=1}^{m-2}\alpha_i\xi_i}y(s)ds \\
&\quad + \sum_{i=2}^{r-1}\int_{\xi_{i-1}}^{\xi_i}\frac{s(1-t)-\sum\limits_{j=i}^{m-2}\alpha_j(\xi_j-t)s+\sum\limits_{j=1}^{i-1}\alpha_j\xi_j(t-s)}{1-\sum\limits_{i=1}^{m-2}\alpha_i\xi_i}y(s)ds \\
&\quad + \int_{\xi_{r-1}}^{t}\frac{s(1-t)-\sum\limits_{j=r}^{m-2}\alpha_j(\xi_j-t)s+\sum\limits_{j=1}^{r-1}\alpha_j\xi_j(t-s)}{1-\sum\limits_{i=1}^{m-2}\alpha_i\xi_i}y(s)ds \\
&\quad + \int_t^{\xi_r}\frac{t[(1-s)-\sum\limits_{j=r}^{m-2}\alpha_j(\xi_j-s)]}{1-\sum\limits_{i=1}^{m-2}\alpha_i\xi_i}y(s)ds \\
&\quad + \sum_{i=r+1}^{m-2}\int_{\xi_{i-1}}^{\xi_i}\frac{t[(1-s)-\sum\limits_{j=i}^{m-2}\alpha_j(\xi_j-s)]}{1-\sum\limits_{i=1}^{m-2}\alpha_i\xi_i}y(s)ds + \int_{\xi_{m-2}}^1\frac{t(1-s)}{1-\sum\limits_{i=1}^{m-2}\alpha_i\xi_i}y(s)ds.
\end{aligned}$$

If $\xi_{m-2} \le t \le 1$, the unique solution (2.3) can be rewritten as

$$\begin{aligned} u(t) &= \int_0^{\xi_1} \frac{s(1-t) - \sum\limits_{j=1}^{m-2} \alpha_j(\xi_j - t)s}{1 - \sum\limits_{i=1}^{m-2} \alpha_i \xi_i} y(s)ds \\ &+ \sum_{i=2}^{m-2} \int_{\xi_{i-1}}^{\xi_i} \frac{s(1-t) - \sum\limits_{j=i}^{m-2} \alpha_j(\xi_j - t)s + \sum\limits_{j=1}^{i-1} \alpha_j \xi_j (t-s)}{1 - \sum\limits_{i=1}^{m-2} \alpha_i \xi_i} y(s)ds \\ &+ \int_{\xi_{m-2}}^{t} \frac{s(1-t) + \sum\limits_{j=1}^{m-2} \alpha_j \xi_j (t-s)}{1 - \sum\limits_{i=1}^{m-2} \alpha_i \xi_i} y(s)ds + \int_t^1 \frac{t(1-s)}{1 - \sum\limits_{i=1}^{m-2} \alpha_i \xi_i} y(s)ds. \end{aligned}$$

Therefore, the unique solution of (2.1)-(2.2) is $u(t) = \int_0^1 G(t,s)y(s)ds$. Lemma 2.2 is proved. □

By Lemma 2.2, the unique solution of BVP (2.1)-(2.2) is $u(t) = \int_0^1 G(t,s)y(s)ds$. Let $w(t) = \int_0^1 G(t,s)h(s)ds$. Obviously $w(t)$ is the unique solution to BVP(2.1)-(2.2) for $y(t) = h(t)$.

LEMMA 2.3. Let $X = C[0,1]$, $K = \{u \in X : u \ge 0\}$. Suppose $T : X \to X$ is completely continuous. Define $\theta : TX \to K$ by

$$(\theta y)(t) = \max\{y(t), w(t)\} \quad \text{for} \ \ y \in TX.$$

where $w \in C^1[0,1]$, $w \ge 0$ is a given function. Then

$$\theta \circ T : X \to K$$

is also a completely continuous operator.

PROOF. The complete continuity of T implies that T is continuous and maps each bounded subset in X to a relatively compact set. Denote θy by $\overline{y}$.

Given a function $h \in C[0,1]$, for each $\varepsilon > 0$ there is a $\delta > 0$ such that

$$\|Th - Tg\| < \varepsilon \quad \text{for} \ \ g \in X, \|g - h\| < \delta.$$

Since

$$\begin{aligned} |(\theta Th)(t) - (\theta Tg)(t)| &= |\max\{(Th)(t), w(t)\} - \max\{(Tg)(t), w(t)\}| \\ &\le |(Th)(t) - (Tg)(t)| < \varepsilon, \end{aligned}$$

we have

$$\|(\theta T)h - (\theta T)g\| < \varepsilon \quad \text{for} \ \ g \in X, \ \|g - h\| < \delta,$$

and so θT is continuous.

For any arbitrarily bounded set $D \subset X$ and any $\varepsilon > 0$, there are y_i, $i = 1, \cdots, m$, such that

$$TD \subset \cup_{i=1}^m B(y_i, \varepsilon),$$

where $B(y_i, \varepsilon) := \{u \in X : \|u - y_i\| < \varepsilon\}$. Then, for any $\overline{y} \in (\theta \circ T)D$, there is a $y \in TD$ such that $\overline{y}(t) = \max\{y(t), w(t)\}$. We choose $i \in \{1 \cdots, m\}$ such that $\|y - y_i\| < \varepsilon$. The fact

$$\max_{0 \le t \le 1} |\overline{y}(t) - \overline{y}_i(t)| \le \max_{0 \le t \le 1} |y(t) - y_i(t)|$$

implies $\overline{y} \in B(\overline{y}_i, \varepsilon)$. Hence $(\theta \circ T)D$ has a finite ε-net and therefore $(\theta \circ T)(D)$ is relatively compact. □

3. EXISTENCE OF SOLUTION

Let $X = C[0, 1]$ and $K = \{u \in X : u(t) \ge 0\}$. Denote by $\| \cdot \|$ the supremum norm on X.

Throughout the rest of the paper we assume that the following conditions are satisfied:

(H1) $0 < \sum_{i=1}^{m-2} \alpha_i < 1$;
(H2) $f : [0, 1] \times [0, \infty) \to R$ is continuous;
(H3) $h(t)$ is a nonnegative measurable function on $[0, 1]$ with $0 < \int_0^1 h(t)dt < \infty$.

If (H1) holds, then $1 - \sum_{i=1}^{m-2} \alpha_i \xi_i > 0$. So $G(t, s) \ge 0$, where $G(t, s)$ given in (2.9) is the Green's function for (2.7)-(2.8). By Hölder's inequality, we have $\int_0^1 |G(t, s)h(s)|ds \le (\int_0^1 |G(t, s)|^2)^{\frac{1}{2}} (\int_0^1 |h(s)|^2)^{\frac{1}{2}} < \infty, \quad t \in [0, 1]$. Let $A = \max_{0 \le t \le 1} \int_0^1 G(t, s)h(s)ds$.

THEOREM 3.1. Suppose there are $r > M > 0$ such that

$$0 < \frac{M}{\min\limits_{0 \le t \le 1} f(t, Mw(t))} = a \le b = \frac{r}{A \max\limits_{\substack{0 \le t \le 1 \\ Mw(t) \le u \le r}} f(t, u)}. \tag{3.1}$$

Then BVP(1.3)-(1.4) has at least one positive solution $u_1(t)$ satisfying

$$0 < Mw(t) \le u_1(t), \quad 0 < t < 1 \text{ and } \|u_1\| \le r$$

if $\lambda \in [a, b]$.

PROOF. Let

$$f^*(t, u) = \begin{cases} f(t, u), & u \ge Mw(t), \\ f(t, Mw(t)), & u \le Mw(t), \end{cases} \tag{3.2}$$

and define $T : K \to X$ by

$$(Tu)(t) = \lambda \int_0^1 G(t, s)h(s)f^*(s, u(s))ds, \quad 0 \le t \le 1. \tag{3.3}$$

Then T is on K a completely continuous operator. If $\theta : X \to K$ is an operator defined by

$$(\theta u)(t) = \max\{u(t), 0\}, \tag{3.4}$$

Lemma 2.3 implies that $\theta \circ T : K \to K$ is also completely continuous.

Take $\Omega = \{u \in K : \|u\| < r\}$. Given $u \in \partial\Omega$, set $I = \{t \in [0,1] : f^*(t, u(t)) \geq 0\}$. Then

$$\begin{aligned}
(\theta \circ T)u(t) &= \max\{\lambda \int_0^1 G(t,s)h(s)f^*(s,u(s))ds, 0\} \\
&\leq \lambda \int_I G(t,s)h(s)f^*(s,u(s))ds \\
&\leq b \max_{\substack{0\leq t\leq 1 \\ 0\leq u\leq r}} f^*(t,u) \int_I G(t,s)h(s)ds \\
&\leq Ab \max_{\substack{0\leq t\leq 1 \\ Mw(t)\leq u\leq r}} f(t,u) \\
&\leq r.
\end{aligned}$$

If there is a $u \in \partial\Omega$ such that $(\theta \circ T)u = u$, then $\theta \circ T$ has a fixed point in $\overline{\Omega}$. On the other hand, if for any $u \in \partial\Omega$, $(\theta \circ T)u \neq u$, it follows that

$$\deg_K\{I - \theta \circ T, \Omega, 0\} = 1,$$

where $\deg_K$ stands for the degree on cone K. Then $\theta \circ T$ has a fixed point in Ω. So in both cases $\theta \circ T$ has a fixed point u_1 in $\overline{\Omega}$.

We claim that

$$(Tu_1)(t) \geq Mw(t), \quad t \in [0,1]. \tag{3.5}$$

If not, then there exists $t_0 \in [0,1]$ such that

$$Mw(t_0) - (Tu_1)(t_0) = \max_{0\leq t\leq 1}\{Mw(t) - (Tu_1)(t)\} = L > 0. \tag{3.6}$$

Obviously, $t_0 \neq 0$. If $t_0 = 1$, we have

$$L = Mw(1) - Tu(1) = \sum_{i=1}^{m-2} \alpha_i[Mw(\xi_i) - Tu(\xi_i)] \leq \sum_{i=1}^{m-2} \alpha_i L < L,$$

a contradiction. So $t_0 \in (0,1)$, and

$$Mw'(t_0) - (Tu_1)'(t_0) = 0.$$

Observe now that we must have

$$Mw(t) > Tu_1(t), \quad t \in [0,1]. \tag{3.7}$$

For if not, then there exists $t_1 \in [0, t_0) \cup (t_0, 1]$ such that

$$Mw(t_1) - Tu_1(t_1) = 0, \quad \text{and} \quad Mw(t) - Tu_1(t) > 0, \quad t \in (t_1, t_0] \text{ or } t \in [t_0, t_1).$$

Without loss of generality we assume $t_1 \in [0, t_0)$. Then for $t \in (t_1, t_0]$,

$$\begin{aligned}
Mw'(t) - (Tu_1)'(t) &= Mw'(t_0) - (Tu_1)'(t_0) - \textstyle\int_t^{t_0} [Mw'(s) - (Tu_1)'(s)]'ds \\
&= \textstyle\int_t^{t_0} h(s)[M - \lambda f^*(s, u_1(s))]ds \\
&= \textstyle\int_t^{t_0} h(s)[M - \lambda f(s, Mw(t))]ds \\
&\leq [M - a \min_{0\leq t\leq 1} f(t, Mw(t))] \textstyle\int_t^{t_0} h(s)ds \\
&= 0,
\end{aligned}$$

i.e., $Mw'(t) - (Tu_1)'(t) \leq 0$, and then

$$Mw(t_0) - Tu_1(t_0) \leq Mw(t_1) - Tu_1(t_1) = 0,$$

a contradiction to (3.6). So (3.7) holds.

However

$$\begin{aligned} Mw(t_0) - (Tu_1)(t_0) &= \int_0^1 G(t_0,s)h(s)Mds - \lambda \int_0^1 G(t_0,s)h(s)f^*(s,u_1(s))ds \\ &= \int_0^1 G(t_0,s)h(s)[M - \lambda f^*(s,u_1(s))]ds \\ &\leq [M - a \min_{0\leq t\leq 1} f(t, Mw(t))] \int_0^1 G(t_0,s)h(s)ds \\ &= 0, \end{aligned}$$

a contradiction to (3.6). So (3.5) hold. Then $(\theta \circ T)u_1 = Tu_1 = u_1$ and $u_1(t)$ is a solution of BVP(1.3)-(1.4). □

THEOREM 3.2. Suppose $f(t,0) \geq 0$, $h(t)f(t,0) \not\equiv 0$ and there is an $r > 0$ such that

$$b = \frac{r}{A \max\limits_{\substack{0\leq t\leq 1 \\ 0\leq u\leq r}} f(t,u)} > 0. \tag{3.8}$$

Then when $\lambda \leq b$, BVP(1.3)-(1.4) has at least one positive solution $u_1(t)$ satisfying

$$0 < \|u_1\| \leq r.$$

PROOF. Let

$$f^*(t,u) = \begin{cases} f(t,u), & u \geq 0, \\ f(t,0) - u, & u < 0. \end{cases} \tag{3.9}$$

The theorem can now be proved by using arguments analogous to that of the proof of Theorem 3.1. □

COROLLARY 3.1. Suppose there is an $M > 0$ such that

$$a = \frac{M}{\min\limits_{0\leq t\leq 1} f(t, Mw(t))} > 0 \tag{3.10}$$

and

$$\lim_{u\to\infty} \frac{\max\limits_{0\leq t\leq 1} f(t,u)}{u} = 0, \tag{3.11}$$

then for $\lambda \geq a$, BVP(1.3)-(1.4) has at least a positive solution u_1 with

$$0 < Mw(t) \leq u_1(t), \quad \|u_1\| < \infty.$$

PROOF. It suffices to show that for any $b > a$, there is an $r > 0$ such that

$$b \leq \frac{r}{A \max\limits_{\substack{0\leq t\leq 1 \\ Mw(t)\leq u\leq r}} f(t,u)}. \tag{3.12}$$

Fix $b > a > 0$. Condition (3.11) implies that there is an $L > 0$ such that

$$\max_{0\leq t\leq 1} f(t,u)/u < \frac{1}{bA}, \quad \text{for } u \geq L,$$

and there exists $r > L$ such that

$$\max_{\substack{0\le t\le 1\\ Mw(t)\le u\le L}} f(t,u)/r < \frac{1}{bA}.$$

Hence

$$\begin{aligned}\max_{\substack{0\le t\le 1\\ Mw(t)\le u\le r}} f(t,u)/r &\le \max\{\max_{\substack{0\le t\le 1\\ Mw(t)\le u\le L}} f(t,u)/r,\ \max_{\substack{0\le t\le 1\\ L\le u\le r}} f(t,u)/r\}\\ &< \max\{\frac{1}{bA},\ \max_{L\le u\le r}[\max_{0\le t\le 1} f(t,u)/u]\}\\ &< \frac{1}{bA},\end{aligned}$$

and in turn (3.12) holds. Applying Theorem 3.1 and by the fact that $b > a$ is arbitrary, the corollary follows. □

COROLLARY 3.2. Suppose condition (3.11) holds and

$$f(t,0) \ge 0, \quad h(t)f(t,0) \not\equiv 0, \quad t \in (0,1),$$

then for any $\lambda \in R$, BVP(1.3)-(1.4) has at least a positive solution u_1 with $0 < \|u_1\| < \infty$.

PROOF. Condition (3.8) can be deduced from (3.11) for any $b > 0$. Hence Theorem 3.2 implies this corollary. □

REMARK 3.1. Theorem 3.1 and Corollary 3.1 can be applied to the case $f \in C([0,1] \times (0,\infty), R)$, i.e., f is singular at $u = 0$.

REFERENCES

1. Ruyun Ma and Nelson Castaneda, Existence of solutions of m-point boundary value problems, *J. Math. Anal. Appl.*, **256**(2001), 556-567.

2. Xiaoming He and Weigao Ge, Triple solutions for second-order three-point boundary value problems, *J. Math. Anal. Appl.*, **268**(2002), 256-265.

3. Youssef N. Raffoul, Positive solutions of three-point nonlinear second boundary value problem, *Electronic Journal of Qualitative Theory of Differential Equations*, **15**(2002), 1-11.

4. Jingli Ren and Weigao Ge, Existence of two solutions of nonlinear m-point boundary value problem, *J. Beijing Inst. of Tech.*, **12**(2003), 97-100.

5. M.A. Krasnoselskii, Positive solutions of operators equations, P. Noordhoff, Groningen, The Netherlands, 1964.

6. R.W. Leggett and L.R. Williams, Multiple positive fixed points of nonlinear operators on ordered Banach spaces, *Indiana Univ. Math. J.*, **28**(1979), 673-688.

7. R. I. Avery and J. Henderson, Twin solutions of boundary value problems for ordinary differential equations and finite difference equations, *Computers Math. Appl.*, **42**(2001), 695-704.