Dynamic Programming with Priority Models for Production Planning

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Abstract

This paper studies a production planning problem. A large soft-drink company in Hong Kong aims at finding a production planning model so as to minimize its inventory build-up and to maximize the demand satisfaction. We formulate the problem by using dynamic programming techniques. We further propose a heuristics model called Priority Model to reduce the computational complexity of this dynamic programming approach. Computational results based on real data sets are given to illustrate the effectiveness of our method.

Key Words: Mathematical modeling, Inventory control, Production planning, Product mix, Dynamic programming, Heuristic model.

1 Introduction

The background of this paper is a production planning and inventory control problem faced by the production plant of a large soft-drink company in Hong Kong. The problem is complex as it involves many inter-related factors, limitations and requirements. These include: large number of product mixes; high volume of sales of products; short delivery lead-time notices; different sizes, types, packaging and processing needs of products; and various machine combinations and capacities. A pressing issue that stands out is the storage space of its central warehouse, which often finds itself in the state of overflow or near capacity. This creates immediate interruptions of production, capital tie-ups and subsequent potential of lost sales. There is a urgent need for the company to study

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the interplay between the storage space requirement and the production lines for better production planning to respond to the overall growing sales demand.

All products and machines under this study are labeled and specified in terms of a product hierarchy structure and a machine hierarchy structure, respectively. At the top-most product hierarchy, there are product categories due to the chemical nature of the products. All products are classified into either low-acid product category or high-acid product category. These two categories lead to different quality control (QC) requirements in terms of incubation needs (from production to QC-ready inventory). Low-acid products have to stay in the central warehouse for a fixed number of days under incubation observation. High-acid products only require on-line QC and are normally ready for next day's shipment. For simplicity, the high-acid products will be considered as requiring one day of incubation observation in this paper. At the bottom-most product hierarchy, there are different packages for the same product flavor, for marketing purposes, into various volume sizes. Finally, all products are labeled as either fast-moving (high sales volume) or slow-moving (low sales volume). Such labeling is useful from both marketing and production planning points of view.

There are individual filling/packaging machines at the lowest level of the machine hierarchy. These machines are (more or less permanently) connected into several machine groups. Machine groups are then "piped" from a fixed number of brewer-sterilizers. The highest hierarchy level specifies the different piping options forming a number of production lines (one line for each brewer-sterilizer). Different piping options hence lead to different combinations of sterilizer feeding different machine groups. Raw materials input into (brewers and then) sterilizers are processed through these production lines to produce the finished goods.

The above product and machine hierarchies do not stand independently of each other. In fact, there is a strict cross-matching relationship between the product hierarchy structure and the machine hierarchy structure. Due to different processing (chemical and packaging) needs of products, they can only be produced on specific machine groups. Hence, a product can be produced on a production line under a given piping option if and only if there are some machines connecting to the line capable of producing this product with all its requirements.

Given such a complicated production system, the objective here is to recommend better production planning to minimize its inventory build-up and to maximize the demand satisfaction. In [3], Chu and Ng have proposed a linear programming model that entails multi-period aggregate optimization to tackle in particular the above two distinguishing (hierarchical) features of the production system. The linear programming model provides individual product-based as well as global pictures of the production planning and inventory control for the operations of the plant. These are useful for decision supports. The issue of production scheduling, which involves additional constraints such as physical line-product assignment and machine non-interference, have been later addressed in [4] using a decomposed linear programming approach. We remark that any actual production automation cannot be implemented without the line specific production scheduling decisions.

This study is an extension of [3]. We aim at addressing such intrinsically combinatorial scheduling problem of high complexity. Here, we model the problem by using dynamic programming (DP) techniques. The physical line-product assignment in the planning horizon can be determined by solving a DP model. Since the number of alternatives at each stage (each day in the planning horizon) are very large (the number of products raised to the power of the number of production lines), the computational complexity of the DP model is enormous. The computation complexity of the real application prohibits the use of DP directly. Several heuristic algorithms for the Wagner-Whitin model have been developed since the early seventies (Florian et al. [8]; Aggarwal and Park [1]; Federgruen and Tzur [7]; Hung et al. [9] and Wagelmans et al [11]). In our study, we introduce a priority model to select several number of alternatives from a set of a large number of alternatives to tackle this combinatorial problem. The set of selected alternatives are determined by solving a linear programming model in the form of multi-period aggregate optimization [2, 5]. We named this heuristic algorithm as Priority Model. As we select only a small number of alternatives at each stage, the computational complexity of the DP model is significantly reduced by this heuristic model. Using the real data from the soft-drink company. The computational results show that the capacities of the production machines can even be reduced to a certain level without incurring lost sales, and therefore the inventory build-up can also be reduced.

The rest of the paper is organized as follows. In Section 2, we define the notations and formulate the mathematical model. Numerical results are presented in Section 3 to demonstrate the effectiveness of our method. Finally, concluding remarks are given in Section 4.

2 Mathematical Model

The following notations are used in our model:

Production Parameters

M	=	number of low-acid products (indexed by $m = 1, \ldots, M$).
N	=	number of high-acid products (indexed by $n = 1, \ldots, N$).
T	=	number of days in the planning horizon (indexed by $t = 1, \ldots, T$).
H	=	number of piping options (indexed by $h = 1, \ldots, H$).
K	=	number of production lines (indexed by $k = 1, \ldots, K$).
U	=	number of packaging sizes (indexed by $u = 1, \ldots, U$).
$G_{h,k}(u)$	=	production capacity of products with the uth packaging in the kth production line under the hth piping option.
$l(m)$	=	length of incubation of the mth low-acid product (in days).
$i_l(m, u, t)$	=	unit inventory holding cost of the mth low-acid product with the uth packaging at time t.
$i_h(n, u, t)$	=	unit inventory holding cost of the nth high-acid product with the uth packaging at time t.
$p(m, u, t)$	=	unit penalty cost of the mth low-acid product with the uth packaging at time t.
$q(n, u, t)$	=	unit penalty cost of the nth high-acid product with the uth packaging at time t.
$D(m, u, t)$	=	forecast demand of the mth low-acid product with the uth packaging at time t.
$E(n, u, t)$	=	forecast demand of the nth high-acid product with the uth packaging at time t.

State Descriptions

$I(m, u, t)$	=	inventory of the mth low-acid product with the uth packaging at time t (including QC-ready and those under incubation).
$J(n, u, t)$	=	inventory of the nth high-acid product with the uth packaging at time t (including QC-ready and those under incubation).
$R(m, u, t)$	=	QC-ready inventory of the mth low-acid product with the uth packaging at time t.
$S(n, u, t)$	=	QC-ready inventory of the nth high-acid product with the uth packaging at time t.

Decision Variables

$X(m, u, h, t)$	=	production level of the mth low-acid product with the uth packaging under the hth piping options at time t.
$Y(n, u, h, t)$	=	production level of the nth high-acid product with the uth packaging under the hth piping options at time t.

The dynamic programming model is formulated by using the Wagner-Whitin backward recursion DP with deterministic products demand [12]. The Optimal Value Function aims to minimize the holding cost (i.e. minimize the inventory build-up) and the backlogging cost (i.e. maximize the demand satisfaction). For simplicity, we assume that the central warehouse has capable additional storage space for those under-incubation inventories. Since the number of states and the number of alternatives are enormous, the computational complexity of the DP computation is extremely high. We develop a heuristic procedure to tackle this computational problem.

2.1 Priority Models

The DP approach is easy to implement, but the computational cost is very expensive. Thus, one of the main challenges here is to tackle the size of the model. In order to achieve our goal, in this section, a priority model is developed for selecting a small number of alternatives from a set of a large number of alternatives so as to reduce the computational complexity of the DP approach. Define $\Phi_{h,t}, t = 1, 2, \cdots, T$ as the decision set consist of possible production plans under piping option h at time t. For each value of a state variable, we prioritize the alternatives in $\Phi_{h,t}, t = 1, 2, \cdots, T$ based on a heuristic rule.

2.1.1 Multi-period Aggregate Linear Programming

To determine the set of "selected" alternatives, we first generate the minimum level of production for each low-acid product denoted by $\tilde{X}(m, u, h, t')$; and $\tilde{Y}(n, u, h, t')$ for each high-acid product, $m = 1, \cdots, M$; $n = 1, \cdots, N$; $t' = t, \cdots, T$; $u = 1, \cdots, U$ by a Multi-Period Aggregate (MPA) linear programming model [2, 5]. It attempts to minimize the holding and penalty cost of the QC-ready inventory from time t to the end of the planning horizon, that is

$$\min \quad \sum_{u=1}^{U} \sum_{t'=t+l(m)}^{T+l(m)} \sum_{m=1}^{M} i_l(m, u, t') \left[R(m, u, t') - D(m, u, t')\right]$$

$$+ \sum_{t'=t+1}^{T+1} \sum_{n=1}^{N} i_h(n, u, t') \left[S(n, u, t') - E(n, u, t')\right], \tag{1}$$

where $i_l(m, u, t')$ (and $i_h(n, u, t')$) is the holding cost per unit inventory for low-acid product (and high-acid product). In this model, our target is to construct the overall production levels, subject to capacity constraints

$$\sum_{m=1}^{M} \tilde{X}(m, u, h, t') + \sum_{n=1}^{N} \tilde{Y}(n, u, h, t') \leq \sum_{k=1}^{K} G_{h,k}(u) \quad (t' = t, \ldots, T, \ u = 1, \ldots, U). \tag{2}$$

The inventory updating constraints of the QC-ready inventories and the demands satisfaction constraints are

$$\begin{cases} R(m, u, t' + l(m)) = I(m, u, t') + \tilde{X}(m, u, h, t') - \displaystyle\sum_{\tilde{t}=t'}^{t'+l(m)-1} D(m, u, \tilde{t}), \\ R(m, u, t') \geq D(m, u, t'), \qquad (m = 1, \ldots, M, t' = t, \ldots, T, u = 1, \ldots, U); \end{cases}$$

$$\begin{cases} S(n, u, t' + 1) = J(n, u, t') + \tilde{Y}(n, u, h, t') - E(n, u, t'), \\ S(n, u, t') \geq E(n, u, t'), \qquad (n = 1, \ldots, N, t' = t, \ldots, T, u = 1, \ldots, U). \end{cases}$$

Since our goal is to estimate the production levels from time t to the remaining days, the time range of the above MPA formulation is restricted from t to T.

2.1.2 Safety Margin (SM)

After solving the MPA model, the next step is to prioritize each alternative by using the generated LP result. MPA is not a scheduling model as the total production level is only restricted by the daily total production capacity of the plant. Thus, the generated decision variables $\tilde{X}(\cdot,\cdot,\cdot,\cdot)$ and $\tilde{Y}(\cdot,\cdot,\cdot,\cdot)$ are only the *planned production planning levels* which may not be achieved at all times with the existing production line setting.

We regard $\tilde{X}(\cdot,\cdot,\cdot,\cdot)$ and $\tilde{Y}(\cdot,\cdot,\cdot,\cdot)$ as our *planned production levels*. If a particular product is selected for production, the foremost information needed to know is the maximum duration (or Safety Margin) of the *planned production planning levels* which can be satisfied by the inventory on-hand plus the production level. Mathematically, it is defined as

$$SM^{low}_{\{\phi_{h,t}(k)\}_{k=1}^{K}}(m,u,t) = \max_{0\le\tau\le T-t}\left\{\tau : I(m,u,t) + \Gamma(m,k,t)G_{h,k}(u) - \sum_{t'=t}^{t+\tau}\tilde{X}(m,u,h,t') \ge 0\right\}$$

for all m and u, and

$$SM^{high}_{\{\phi_{h,t}(k)\}_{k=1}^{K}}(n,u,t) = \max_{0\le\tau\le T-t}\left\{\tau : J(n,u,t) + \Lambda(n,k,t)G_{h,k}(u) - \sum_{t'=t}^{t+\tau}\tilde{Y}(n,u,h,t') \ge 0\right\}$$

for all n and u, where $\{\phi_{h,t}(k)\}_{k=1}^{K}$ is one of the alternatives in the decision set $\Phi_{h,t}$. Hence, there are totally $[(M+N)U]^K$ sets of *Safety Margin* (SM) for each state in the DP model. Moreover, there are two indicating variables

$$\Gamma(m,k,t) = \begin{cases} 1, & \phi_{h,t}(k) = m, \\ 0, & \text{otherwise.} \end{cases}$$

and

$$\Lambda(n,k,t) = \begin{cases} 1, & \phi_{h,t}(k) = M+n, \\ 0, & \text{otherwise.} \end{cases}$$

Here $\Gamma(m,k,t)$ indicates that low-acid product m is selected to produce by production line k at time t. Similarly, where $\Lambda(n,k,t)$ indicates that high-acid product n is selected to produce by production line k at time t

For computational convenience, if the kth production line cannot produce the uth packaging size, then we set

$$SM^{low}_{\{\phi_{h,t}(k)\}_{k=1}^{K}}(m,u,t) = SM^{high}_{\{\phi_{h,t}(k)\}_{k=1}^{K}}(n,u,t) = \infty, \quad \forall m,n.$$

2.1.3 SM Zero-Count (SZ)

We seek for a production plan that can fulfill the requirement of the MPA as much as possible, and a product with a high value of SM indicates that the production plan is favorable in this sense. On the contrary, a low SM value implies that the production plan is inadequate for that product. As a result, we would prefer a production setting that can maximize all the SM values. The first measures SM Zero-Count is develop base on this concept. Mathematically, SM Zero-Count (SZ) is defined as the number of elements in the following set:

$$\left\{SM^{\{\cdot\}}_{\{\phi_{h,t}(k)\}_{k=1}^{K}}(\cdot,\cdot,t) : SM^{\{\cdot\}}_{\{\phi_{h,t}(k)\}_{k=1}^{K}}(\cdot,\cdot,t) = 0\right\}.$$

For each decision $\{\phi_{h,t}(k)\}_{k=1}^{K}$, we have $(M+N)$ SZ values. If the production level of a specific setting cannot accomplish even a day's *planned production planning levels* of any one product (i.e., $SM^{low}_{\{\phi_{h,t}(k)\}_{k=1}^{K}}(\cdot,\cdot,t) = 0$ or $SM^{high}_{\{\phi_{h,t}(k)\}_{k=1}^{K}}(\cdot,\cdot,t) = 0$), this production setting will be regarded as a poor performance decision.

2.1.4 SM Standard Deviation (SD)

The following step is a tie-breaking rule for decisions with the same SZ values using the concept of standard deviation. We favor SM values with low variations in the sense that this production plan provides a fair product satisfaction level among different products. Mathematically speaking, we have the mean and the standard derivation of SM given respectively by

$$SA(\{\phi_{h,t}(k)\}_{k=1}^{K}) = \frac{\sum_{u=1}^{U}\left[\sum_{m=1}^{M} SM^{low}_{\{\phi_{h,t}(k)\}_{k=1}^{K}}(m,u,t) + \sum_{n=1}^{N} SM^{high}_{\{\phi_{h,t}(k)\}_{k=1}^{K}}(n,u,t)\right]}{U(M+N)}$$

and

$$SD(\{\phi_{h,t}(k)\}_{k=1}^{K}) = \sqrt{\frac{\sum_{1}^{U}\sum_{1}^{M+N}[SM^{\{\cdot\}}_{\{\phi_{h,t}(k)\}_{k=1}^{K}}(\cdot,\cdot,t) - SA(\{\phi_{h,t}(k)\}_{k=1}^{K})]^2}{U(M+N)-1}}.$$

We use a two product example to illustrate the reason of choosing a small SD. Given a production problem with one low-acid product, one high-acid product, and one production line. At each planning horizon, the plant is going to produce either one of the soft-drink products. Presently, the product levels of the low-acid product and the high-acid product could satisfy 3 and 1 time-horizon demands respectively. The first alternative in $\Phi_{1,t}$ is to produce the low acid product which would bring SM^{low} to 6. The second option is to produce the high acid product turns SM^{high} to 3. The existing product levels would satisfy at least one day's demand thus SZ for both options are zero. We then move to evaluate the SD values. SA value for both options is 3.5. However, the standard deviation for the second option is 0.7. Apparently, producing the high-acid products would bring a higher demand satisfaction level for both products in the coming time period.

2.1.5 SZ Relative Values (SR)

Nevertheless, several decisions would probably attain the same $SZ(\centerdot)$ and $SD(\centerdot)$ values, so the next step is to find a further way to prioritize them. Actually, we expect these two production decisions to generate similar outcomes. Nonetheless, it is still worthy to prioritize these two decisions by observing their corresponding $SZ(\centerdot)$ values. An idea is that if the production level cannot even fulfill a day's *planned production planning levels* ($\tilde{X}(\centerdot,\centerdot,\centerdot,\centerdot)$ or $\tilde{Y}(\centerdot,\centerdot,\centerdot,\centerdot)$), then we need to know its relative proportion (denoted as $SR(\centerdot)$) of this *planned production planning levels* to the total demand. The mathematical definition for $SR(\centerdot)$ is given as follows:

$$SR(\{\phi_{h,t}(k)\}_{k=1}^{K}) = \sum_{m=1}^{M}\sum_{u=1}^{U}\mathcal{G}_{\{\phi_{h,t}(k)\}_{k=1}^{K}}(m,u,t) + \sum_{n=1}^{N}\sum_{u=1}^{U}\mathcal{F}_{\{\phi_{h,t}(k)\}_{k=1}^{K}}(n,u,t),$$

where

$$\mathcal{G}_{\{\phi_{h,t}(k)\}_{k=1}^{K}}(m,u,t) = \begin{cases} \frac{\tilde{X}(m,u,h,t)}{\sum_{t'=t}^{T+l(m)} D(m,u,t')}, & \text{if} \quad SM^{low}_{\{\phi_{h,t}(k)\}_{k=1}^{K}}(m,u,h,t) = 0, \\ 0, & \text{otherwise} \end{cases} \qquad \forall m,u;$$

and

$$\mathcal{F}_{\{\phi_{h,t}(k)\}_{k=1}^{K}}(n,u,t) = \begin{cases} \frac{\tilde{Y}(n,u,h,t)}{\sum_{t'=t}^{T+1} E(n,u,t')}, & \text{if} \quad SM^{high}_{\{\phi_{h,t}(k)\}_{k=1}^{K}}(n,u,h,t) = 0, \\ 0, & \text{otherwise} \end{cases} \qquad \forall n,u.$$

A small $SR(\centerdot)$ indicates that the demands of the products which have a zero $SM(\centerdot)$ are in comparatively lower overall requirements. Therefore we would prefer the decision plan with a small $SR(\centerdot)$ value.

2.1.6 The Heuristic Procedures

The decision set $\Phi_{h,t}$ at each state is an important component in the DP model. As mentioned in Section 2, the number of alternatives in $\Phi_{h,t}$ is $(N+M)^K$. This leads to the high computational complexity of the DP model. It is clear that if we can reduce the number of alternatives in $\Phi_{h,t}$, then the computational complexity of the DP model can be significantly reduced. In the previous subsection, we have developed a procedure on how to generate priority tables. In the following we give a summary of our heuristic algorithm for generating the alternatives in $\Phi_{h,t}$, when we have the information of the current state at time t ($\mathcal{I}_t$ and $\mathcal{J}_t$) and the demand of products from time t to T.

Priority Generation Steps (PGSs):

Step 1: Solve the multi-period aggregate linear programming model for $\{\tilde{X}(m,u,h,t')\ \forall m,u\}_{t'=t}^{T}$ and $\{\tilde{Y}(n,u,h,t')\ \forall n,u\}_{t'=t}^{T}$; goto **Step 2**.

Step 2: Compute $SM^{low}_{\{\phi_{h,t}\}_{k=1}^{K}}(m,u,t)$ for all m, u, and $SM^{high}_{\{\phi_{h,t}(k)\}_{k=1}^{K}}(n,u,t)$ for all n, u at time t; goto **Step 3**.

Step 3: Taken as a ranking based on dictionary order, SZ-SD-SR(all ascending).

Step 4: Select the first $\tilde{d}$ decision plans in the priority list each with different production plan and generate $\tilde{d}$ state(s) at time $t+1$ with respect to these $\tilde{d}$ different decision plans.

After computing all the state values, the DP model becomes a decision tree. Simply find the best path from time 1 to T to finish the DP and heuristics computations.

2.1.7 Number of Alternatives

The number of alternatives, $\tilde{d}$, is a user's decision parameter, which determines the number of choices in the decision set $\Phi_{h,t}$ at each state. That is the size of the final decision tree. The larger the value of $\tilde{d}$, the better the solution will be, but also the higher the computational cost. Thus, before any numerical computation, users need to balance between time and accuracy.

3 Numerical Results

This section presents the production plan determined by the heuristic method. Recall that the soft-drink company has a huge number of soft-drink products, and some of those have seasonal oversea demands. Therefore, we confine ourselves to local products with continuous yearly demands.

According to the company's production plant settings, the daily operation hours for the production lines and the machine maintenance and production set-up time are fixed with the following parameters. The production horizon is of length $T = 12$ (days). The production plan consists of $K = 5$ production lines, and there are $H = 9$ sets of piping options. Totally 15 different products, of which $N = 8$ high-acid products and $M = 7$ low-acid products, are available. There are a maximum of $U = 3$ volume sizes for each product (which implies that some products have two different volume sizes, and some have one volume size only). The incubation period for all low-acid products is $l(m) = 6, \forall m = 1, \ldots, 7$. For simplicity of presentation, the unit holding cost and penalty cost across all products are assumed to be equal. Moreover, as the company prefers overstocking over under-stocking, the unit penalty cost is assumed to be five times the unit holding cost which we believe the ratio is strong enough for demonstration. Mathematically,

$$5i_l(m,u,t) = 5i_h(n,u,t) = p(m,u,t) = q(n,u,t).$$

The initial inventory levels for all the low-acid products is sufficient to cover at least the

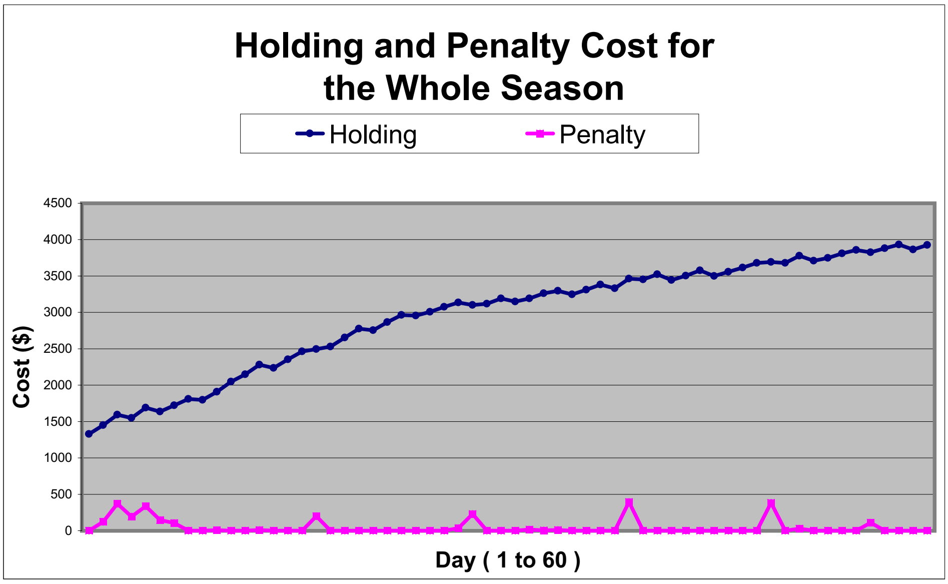

Figure 1: Holding and Penalty Cost Seasonally

first six days demand (i.e. the incubation period). At the same time, all the first day high-acid product demands can be satisfied by their initial inventory.

In the following discussion, we will select one piping option to illustrate the details of the computational result. An important observation for the cost function is the extremely low penalty cost which is around 5% of the accumulated total cost (see Figure 1). We have achieved part of our first objective, i.e., to maximize the company's demand satisfaction.

After checking inventory levels of individual products, we find that the accumulated inventory levels are from those overstocking slow-moving products (See Figure 2). Since the demands for the slow-moving products are relatively low, just one day's production level can satisfy the whole planning horizon's requirement. To fully see the model's performance for a longer period, we further extend the study to five consecutive planning horizons (called cycles) and the results are show in Figure 1. It is found that the cost for the whole season is gradually increasing. After the first cycle's production, the accumulated slow-moving products are sufficiency enough to cover the following one to two cycles' demands. Since the demands for fast-moving products are relatively high, most of the fast-moving products will be awarded higher Safety Margins. Thus, the production plan will tend to produce excessive amount of these products. This phenomenon can be seen in Figure 2 as well. The holding cost for the fast-moving product is steadily increasing.

The accumulated production levels and accumulated demand for four selected products are plotted in Figure 3 (a) Fast-moving Low-acid Product, (b) Slow-moving Low-acid Product, (c) Fast-moving High-acid Product and (d) Slow-moving High-acid Product. It is interesting to note that the performance for the fast-moving high-acid product is the best. It can be explained by the fact that high-acid products only require spending one night at the central warehouse, ready for next day's shipment. Also, as high demand products, the production selection ranking for them are relatively high. Although the fast-

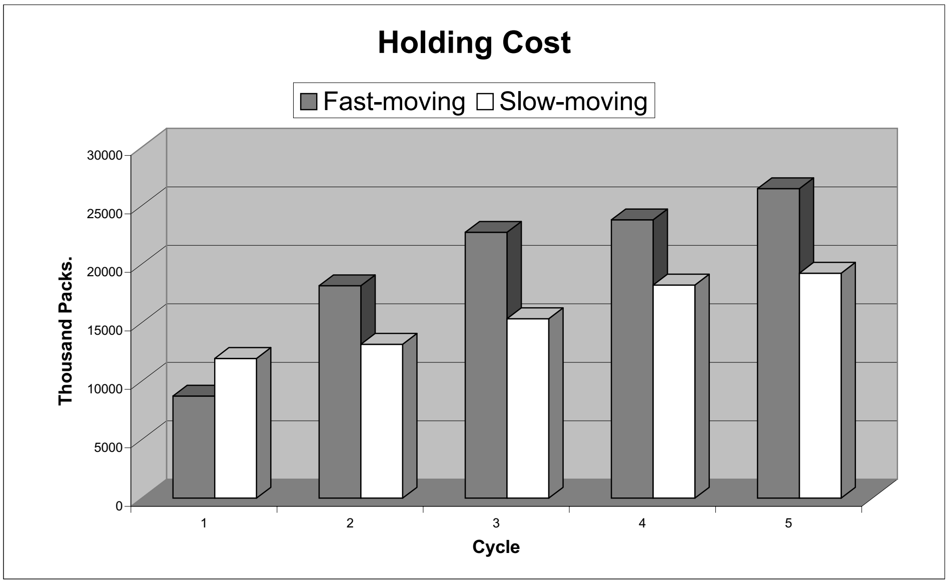

Figure 2: Holding cost for All Types of Products

moving low-acid products also share the same advantage, the accumulated inventory level is comparatively large because of the long incubation requirement. The performances for the two slow-moving products are quite similar. The only difference is that the low-acid products sometimes get into the under-stocking situation. This is also the result of the long incubation period. Having a low demand level, the slow-moving product always receives a lower ranking in the Priority List. This would not be changed unless the under-stocking situation reaches an unacceptable level. Then, the linear programming model would tend to signal this and generate a sufficient amount of minimum level to it. However, a low-acid product is not immediately usable unless it has passed the incubation period. It means that the production will often be too late leading to backlogs. Luckily, the demands for the low-acid slow-moving products account for a relatively low proportion.

In general the results given by this heuristic model are very good. Especially for those products of high sales, the carried inventory fits the demand curve rather well. Moreover, in the long-run, the under-stocking situation is rare and the rate of increase of holding cost drops gradually.

4 Conclusions

We have described a production planning and inventory control model in a soft-drink company. A DP model with heuristic algorithm for its computation is developed and tested. The production planning problem is complicated in three respects: a complex capacitated production facility, a huge variety of products with different demands, and a limitation in the central warehouse storage.

The developed heuristic algorithm performs quite well. Estimated by the synthetic data sets, the heuristic algorithm produces an acceptable solution for a combinatorial

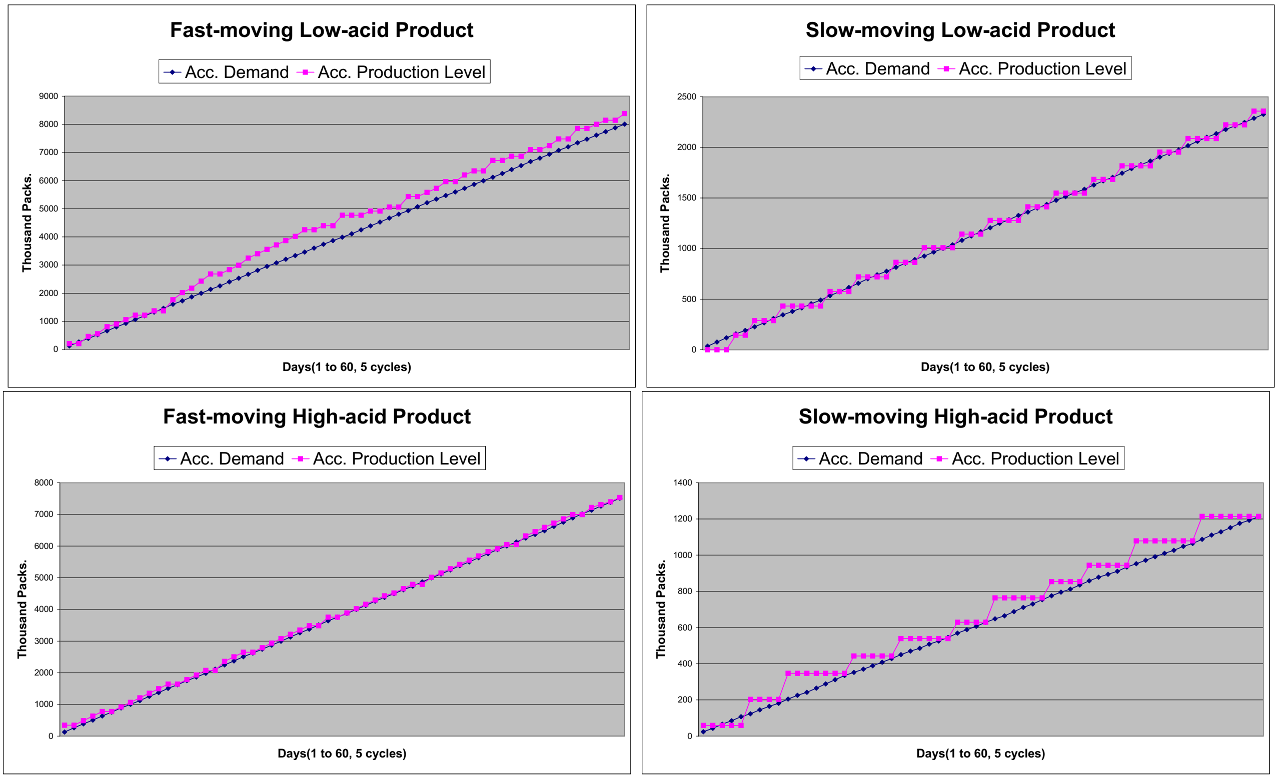

Figure 3: Accumulated Production Level vs Accumulated Demand (a) Selected Fast-moving Low-acid Product (b) Selected Slow-moving Low-acid Product (c) Selected Fast-moving High-acid Product (d) Selected Slow-moving High-acid Product

DP model within 18%-error in less than 0.01% of the computational time for generating the optimal solution. Apart from the fast computational time, another advantage of the developed heuristic algorithm is that it is easy to explain and implement. It only requires a programming language that is capable to solve the linear programming model as a subroutine LP model solver.

Our heuristic algorithm is also very flexible. The solution can be easily improved by expanding the range of the alternatives in the *Priority List*. For users who weigh more on the computational efficiency, using a small number of alternatives leads instead to a smaller DP model.

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