Automorphic Galois representations

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Abstract

This talk is based on joint work with Richard Taylor, in which we generalize to dimension n ¿ 2 the techniques and results of the article of Taylor and Wiles, "Ring-theoretic properties of certain Hecke algebras. Our theorem is roughly as follows. Let K be an imaginary quadratic field and k a finite field of characteristic ℓ, with ℓ split in K, and let $r : Gal(\overline{\mathbf{Q}}/K) \to GL(n,k)$ be an absolutely irreducible representation that can be realized in the cohomology of a Shimura variety uniformized by the unit ball in $\mathbf{C}^{n-1}$, corresponding to a congruence subgroup of the unitary group G of a division algebra over K with an involution of the second type. We suppose that r comes from an automorphic representation π of G whose component at some p-adic place v, with p different from ℓ, is supercuspidal. Then every deformation r' of r to an ℓ-adic representation of $Gal(\overline{\mathbf{Q}}/K)$, which is crystalline at ℓ with fixed Hodge-Tate weights, is realized in the cohomology of Sh. In particular, the Dirichlet series $L(s,r')$ is entire and satisfies a functional equation. In the current state of knowledge, this can only be proved under some additional technical hypotheses on π and r.