

Vertex algebras and Hilbert schemes of points on surfaces

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Abstract

There is a surge of research on Hilbert schemes of points on surfaces since the discovery of the infinite dimensional Heisenberg algebra action on the cohomology groups of the Hilbert schemes of points on surfaces by Nakajima (Ann. Math. 1997). The interplays between the Heisenberg action and the cup product on the cohomology groups of the Hilbert schemes seems to be the most interesting part of this research. Lehn (Invent. Math. 1999) found the Virasoro algebra actions on the cohomology groups of the Hilbert schemes via the study of the commutation relations between the Heisenberg operators and the cup product by the boundary of the Hilbert schemes. Working with Qin and Wang, we studied the relations between Chern classes of tautological vector bundles on the Hilbert schemes and some vertex operators which are generalizations of Heisenberg and Virasoro operators. We were able to find two sets of generators for the cohomology ring structures of the Hilbert schemes. One set of such generators was found before by Ellingsrud and Stromme, Beauville and Markman for the projective plane, surfaces with Kodaira dimension negative, and K3 surfaces respectively. Their methods depend upon the geometry of the surfaces and seem not to work for other type of surfaces such as surfaces of general type. Our results were used recently by Lehn and Sorger, among results of Nakajima and Lehn, to determine the cohomology ring of the Hilbert schemes of points on K3 surfaces via the datum from symmetric groups and the lattice of the cohomology groups of the K3.