Degeneration of moduli spaces of vector bundles

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Abstract

When a smooth curve of genus g degenerates to a node curve, the associated moduli space $M(r,d)$ of semistable vector bundles will degenerate to a moduli space $T(r,d)$ of semistable torsion free sheaves. In an earlier work (J. of Algebraic Geom. 93, 2000), we studied the singularitiy and normalization of this singular moduli and its natural subvarieties, and the results were used to prove the so called "Factorization of generalized theta functions" for higher rank, which generalized the result of rank two by Narasimhan and Ramadas (Invent. Math. 114, 1993). Before our work, it was thought "...generalization to higher rank seems out of reach" (A. Beauville, "current topics in complex algebraic geometry", 1995). The reviewer refered to it as a remarkable paper (Math. Review 1752012). On the other hand, the moduli space $M(r,L)$ of semistable vector bundles with fixed determinant L is a closed subvariety of $M(r,d)$, its degeneration $T(r,L)$ will be a subvariety of $T(r,d)$. However, degeneration of $SL(r)$-bundles posts difficults (G. Faltings, Math. Ann. 304, 1996), thus a interesting question is to define $T(r,L)$ as a moduli space of some "degenerated $SL(r)$-bundles", and determine its singularity (maybe more challenging as Faltings belived). We will give a description of the closed points of $T(r,L)$ and its singularity when the curve degenerates to a reducible curve with two smooth irreducible components intersecting one node.