Exterior Differential Systems and the Calculus of variations

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Abstract

Although the classical formalism of the calculus of variations is a venerable subject about which it seems not much can be said, it has some major drawbacks when dealing with intrinsic geometric problems. In 1983, P.A. Griffiths developed a new formalism of the subject via the language of exterior differential systems. This new formalism is, while in greater generality than customary, particularly effective for differential geometric problems, and it sheds new light on even the classical Lagrange problem.

In this talk, we shall discuss the generalization of Griffiths' formalism to the case of several independent variables, and by studying a spectral sequence associated to the infinite prolongation of the Euler-Lagrange equations associated to a general variational problem, we extend the classical notion of Noether symmetries to "higher order Noether symmetries" and from these we compute all higher order conservation laws of the Euler-Lagrange equations.