Eckart Viehweg

Chern classes of Gauß-Manin Bundles

Abstract

Let $\bar{S}$ be a smooth complex variety, T a normal crossing divisor and $S = \bar{S} \setminus T$. For a proper smooth family $f : X \to S$ the bundles $\mathcal{H}^i := R^i f_* \Omega^\bullet_{X/S}$ are endowed with the Gauß-Manin connection, and thereby, e.g. by Chern-Weil theory, their Chern character in de Rham cohomology is trivial. If the local monodromies around the components of T are uni-potent, the same remains true for the Chern character of the Deligne extension $\overline{\mathcal{H}}^i$.

Mumford remarked that the Grothendieck-Riemann-Roch theorem applied to a family of semistable curves $\bar{f} : \bar{X} \to \bar{S}$, extending f, yields vanishing of the Chern character of $\overline{\mathcal{H}}^1 = R^1 \bar{f}_* \Omega^\bullet_{\bar{X}/\bar{S}}(\log(\bar{X} \setminus X))$ in the Chow ring $CH^{\bullet>0}(\bar{S}) \otimes \mathbb{Q}$. This lead Hélène Esnault to wonder, whether this result extends to families of higher dimensional manifolds. If the fibres of f are abelian varieties of dimension g, van de Geer proved that $\mathrm{ch}(\mathcal{H}^1)$ is torsion in $CH^{\bullet>0}(S)$, and using Mumford's result for curves it is easy to extend this result to $\mathrm{ch}(\overline{\mathcal{H}}^1) \in CH^{\bullet>0}(\bar{S})$ for $g \leq 3$ (van de Geer) and $g = 4, 5$ (Iyer).

A recent article (joint work with Hélène Esnault) gives a solution for arbitrary families of semi-stable abelian varieties:

Let $\mathcal{H}^1$ be a variation of polarized pure Hodge structures of weight 1, with unipotent local monodromies along the components of T, and let $\overline{\mathcal{H}}^1$ be its Deligne extension. Then $\mathrm{ch}(\overline{\mathcal{H}}^1) \in CH^0(\overline{S}) \otimes \mathbb{Q}$..