Estimates for the $\overline{\partial}$-Neumann problem and nonexistence of Levi-flat hypersurfaces in $\mathbb{C}P^n$

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Abstract

Let Ω be a *pseudoconvex* domain with C^2-smooth boundary in $\mathbb{C}P^n$, $n \geq 2$. We prove that the $\overline{\partial}$-Neumann operator N exists for on Ω. Furthermore, there exists an $\epsilon_0 > 0$ such that the $\overline{\partial}$-Neumann and related operators are regular in the Sobolev space $W^\epsilon(\Omega)$ for $\epsilon < \epsilon_0$. We use the estimates to prove the nonexistence of $C^{2,\alpha}$ real Levi-flat hypersurfaces in $\mathbb{C}P^n$. This improves earlier results by Y.-T. Siu who proved the nonexistence of C^8 real Levi-flat hypersurfaces in $\mathbb{C}P^n$. We also show that there exist no non-zero L^2-holomorphic $(p,0)$-forms on any *pseudoconcave* domain with C^2 boundary in $\mathbb{C}P^n$ with $p > 0$.