



Institute of Mathematical Research
HKU



Department of Mathematics
HKUST



Department of Mathematics and IMS
CUHK

Hong Kong Geometry Colloquium

June 5, 2004 (Saturday)

Room 517, Meng Wah Complex, HKU

10:00 – 11:00am

Professor Kang Zuo

CUHK, Hong Kong and Universität Mainz, Germany

Geometry and Arithmetic of Moduli Stacks of Polarized Manifolds

11:00 – 11:20am

Tea Break

11:20am – 12:20pm

Professor Yum-Tong Siu

William Elwood Byerly Professor, Harvard U., USA and
C.V. Starr Visiting Professor, HKU

Some Open Problems and Techniques in Complex and Algebraic Geometry

This meeting is hosted by the Institute of Mathematical Research, HKU.
Abstracts are posted on our website < <http://www.hku.hk/math/imr/> >.

All are Welcome

Geometry and Arithmetic of Moduli Stacks of Polarized Manifolds (A joint programme with Eckart Viehweg)

Abstract: Let M_h denote the moduli scheme of n -dimensional minimal polarized complex manifolds with Hilbert polynomial h . Assume that $\varphi : U \rightarrow M_h$ is induced by a family $f : V \rightarrow U$, for $U = Y \setminus S$ a quasi-projective manifold ($\iff U \rightarrow M_h$ factors over the moduli stack), and let $\mathbf{H} = \text{Hom}((Y, U), (\bar{M}_h, M_h))$ denote the scheme parametrizing all $\varphi : Y \rightarrow \bar{M}_h$ with $\varphi(U) \subset M_h$. It has been shown \mathbf{H} is of finite type (Viehweg-Z).

Two General Questions:

I) If φ is finite, generically finite, or non-constant, what can be said about U ?

II) φ finite: Which properties of U are reflected in properties of a general fibre F of f , and vice versa?

To Question **I)** one has shown that U is algebraic hyperbolic (Migliorini, Kovacs, Viehweg-Z) and is Brody-hyperbolic for canonically polarized case (Viehweg-Z). As for Question **II)** one knows that if the iterated Kodaira-Spence map along a family $f : X \rightarrow U$ is not zero, then this family is rigid (Viehweg-Z, Liu-Todorov-Yau-Z). In his thesis Zhang Yi has shown if a family has a maximal degenerated fibre then it is rigid.

In this programme we propose to study the further geometric and arithmetic properties on M_h .

a) Study the geometry of the moduli scheme \mathbf{H} .

Conjecture I If M_h is the moduli space of Calabi-Yau 3-fold, then \mathbf{H} is a locally Hermitian symmetric space of $\text{SU}(1, n)$ or $\text{Spin}(2, m)$.

b) Study the arithmetic properties of the Mumford-Tate group of the generic fibre F of the universal family $f : X \rightarrow \mathbf{H} \times U$.

Conjecture II If M_h is the moduli space of Calabi-Yau n -folds, then $H^n(F, \mathbb{Q})$ is a \mathbb{Q} -component of a direct sum of tensor product of polarized \mathbb{Q} -Hodge structure of weights $< n$. If $n = 3$, then $H^n(F, \mathbb{Q})$ is a motive cohomology.

c) Find the right notion of (strictly) maximal Higgs field on families over higher dimensional base, such that if a family $f : X \rightarrow U$ admits a strictly maximal Higgs field then $f : X \rightarrow U$ is a modular family of Hodge-type.

Conjecture III There exist no families of minimal polarized manifolds of positive Kodaira dimension admitting strictly maximal Higgs field. In particular, a Shimura curve of Mumford-type cannot be contained in a moduli stack of minimal polarized manifolds of positive Kodaira dimension.

In my lecture I shall concentrate myself on one of the project and give a brief report on the recent progress.

Some Open Problems and Techniques in Complex and Algebraic Geometry

Professor Yum-Tong Siu

William Elwood Byerly Professor, Harvard U., USA and
C.V. Starr Visiting Professor, HKU

Abstract

Will give a survey of motivations, results, techniques, open problems, and possible approaches to the solutions of open problems in the area of analytic methods in complex and algebraic geometry.