

Abstracts

Beda Chan, Statistics and Actuarial Science, HKU

The Ruin Probability of the Secondary School Places Allocation (SSPA) Lottery

Since 1978, the Education and Manpower Bureau (EMB) in Hong Kong have been controlling the placement of Primary 6 students into Secondary 1. Over time, the procedure has evolved into an exercise named the Secondary School Places Allocation, which is in fact a lottery. In April-July 2004, for example, this rite of passage is dished out to 81,801 such students. In this paper, we investigate the ruin probability of the EMB holding a portfolio of about eighty thousand policies with annual claims under the current SSPA regime.

Ngai-Hang Chan and Michael Wong, Statistics, CUHK

Data Mining of Resilience Indicators

In recent years, the Asian-Pacific region has experienced several financial pitfalls including speculative attacks in 98 and the SARS outbreaks in 2003. Financial stresses of this nature are unanticipated, and not all of the perils can be predicted through examining market information and macroeconomic indicators. An early warning systems adopted by the IMF may not be able to predict future financial crisis for all possible scenarios. In fact, shocks come with different forms. To supplement the EWS, this paper proposes a data mining framework to measure the resilience of an economy. The resilient framework does not predict crisis, but rather to assess current state of health of an economy and hence its ability to withstand financial shocks should one occur. The framework is based on a feedback system of two stages. The first stage assigns resilience score to each economy based on a fuzzy logic scoring scheme built on ambiguous reasonings of expert opinions. The second stage uses the classification and regression tree approach to estimate thresholds for each economic indicators and examines the quality of the fuzzy score. The result from the second stage is then passed back to the first stage as a feedback. The final result is obtained until the system reaches its equilibrium state. The proposed resilience framework is applied to the external sector of several countries to illustrate its applicability.

Wai-Sum Chan, Statistics and Actuarial Science, HKU

An Actuarial Analysis of Long-Term Care Insurance Demand in Hong Kong

Hong Kong is a fast aging society. More than 775,000 citizens (around 11 % of the total population) in Hong Kong were aged 65 or above in 2002 and it is expected the figure will increase to 2.2 million (or 24 % of the total) in 2031. In this paper, we provide an actuarial projection of the demand of LTC services in Hong Kong using the U.S. National Long-Term Care Surveys (NLTCs) data. The need for LTC in Hong Kong will be growing drastically at older ages in the next few years: around 10 % for elderly aged between 65 and 74 and the proportion would mount to 60 % for the elderly at age 85 or above. Using a Markov Chain model, an individual's (age-gender specific) future LTC expenditures in Hong Kong have been projected. The net actuarial present values of total future LTC costs are expected to be HK83,000 and HK160,000 for males aged 65 and females aged 65, respectively. Alternative funding strategies for these liabilities are discussed. A simple private insurance scheme is chosen to illustrate the actuarial calculations. The results could be useful for funding and planning LTC services in Hong Kong.

Ka Chun Cheung, Statistics and Actuarial Science, HKU

Optimal Stopping Behavior of Equity-Indexed Annuity with Regime Switching

In this paper, we will study the optimal behavior to surrender an Equity-Indexed Annuity (EIA) product in a discrete-time setting. The return credited to the EIA account will depend on the return of some underlying index. We assume that the market environment will switch among different regimes in a Markovian way, and the return of the index will have different distributions in different regimes. Assuming a CRRA preference, we have obtained the optimal stopping rule. Properties about the optimal stopping behavior, in particular the effect of regime switching, are examined.

Kequan Ding, Department of Mathematics, Graduate School of Chinese Academy of Sciences

Reference Price in Real Estate Pricing

A reference price in asset pricing is supposed to be a gauge for consumers. In this talk, we explore a logical dilemma a consumer may encounter in asset pricing when using reference price. We use an example from real estate pricing to demonstrate the underlying mechanism and its financial consequences.

Hans U. Gerber, Ecole des Hautes Etudes Commerciales, University of Lausanne

On Optimal Dividends: From Reflection to Refraction

The problem goes back to a paper that Bruno de Finetti presented to the International Congress of Actuaries in New York (1957). In a stock company that is involved in risky business, what is the optimal dividend strategy, that is, what is the strategy that maximizes the expectation of the discounted dividends (until possible ruin) to the shareholders? Jeanblanc-Picque and Shiryaev (1995) and Asmussen and Taksar (1997) solved the problem by modeling the income process of the company by a Wiener process and imposing the condition of a bounded dividend rate. Here we present some down-to-earth calculations.

Marc Goovaerts, CRIS, Actuarial Science, K.U. Leuven

Actuarial Risk Measures

In the present talk we provide an overview of the recent developments in actuarial risk measures. They originated as insurance principles, see Markowitz (1959), and were repackaged as premium calculation principles, see Bühlmann (1970). In the last decade, they were repackaged once more as risk measures. Risk measures can be characterized by axioms that may be different for different applications. An overview of the meaning of some of the axioms is given in the framework of capital allocation, economic capital, reinsurance and rating. Their relevance in the framework of risk management is explained. We stress the importance of the relation of risk measures with the dependence structure of the risks under consideration. By means of the Esscher transform a vector of comonotonic random variables is constructed from a vector of independent random variables and a risk measure that is additive for independent risks is transformed into a risk measure that is additive for comonotonic risks, allowing a straightforward characterization. This enables us to deduce a new characterization of the so-called mixture of Esscher principles. Several other characterizations of risk measures are given. We indicate that in actuarial practice the Markowitz ideas are often misunderstood. In addition the influence of the time horizon on risk measures is examined.

Zhenting Hou, School of Mathematical Sciences and Computing Technology, Central south University

Markov Skeleton Processes and Their Applications to Risk Theory

A stochastic process $X(t)$ is called a Markov skeleton process(MSP) if it has the Markov property on a sequence of stopping times $\tau_n \uparrow \infty$. We have obtained the backward and forward equations that determine the probability distributions of these processes. The usual Markov process, semi-Markov process, deterministic Markov process (proposed by Mark Davis) can be regarded as a special case of MSP. In $GI/G/n$ queues, queuing network, repairable system using serial and/or parallel connection, storage system, stock option system, futures system and banking system of difference between deposits and loans, etc, we find that most of the stochastic processes that occur are MSP or can be converted to MSP. For example, risk process

$$X(t) = u_t + \int_0^t C(y(s))ds - \sum_{k=1}^{N_t} Z_k + W(t)$$

is a MSP with discontinuous points of $N(t)$ as its skeleton time sequence, where $y(t)$ is a finite state semi Markov process, $N(t)$ is a renewal process, Z_k is a random variable with general distribution, $W(t)$ is a Brownian motion. The purpose of this paper is to give the basic theory of MSP and show several applications of MSP to the above-mentioned fields.

Yijun Hu, School of Mathematics and Statistics, Wuhan University

Asymptotic Ruin Probabilities for Discrete Time Risk Models with Heavy-Tailed Claims

Consider a discrete time risk model $U_n = (U_{n-1} + X_n)(1 + I_n) - Y_n, n = 1, 2, \dots$, where $U_0 := M > 0$ is the initial reserve of an insurance company, X_n the total amount of premiums, Y_n the total amount of claims, I_n the interest rate and U_n the reserve at time n . Define the time of ruin by $\tau_M := \inf\{n \geq 1; U_n < 0\}$. Assume that $\{Y_n\}$ are heavy-tailed. Our main objective is to give reasons for the asymptotic estimate $P(\tau_M < \infty) \approx M^{-\lambda}$ where λ is a specific positive parameter. A more general risk model from Nyrhinen (1999) is also discussed, and similar asymptotic estimate for ruin probabilities is given.

Tse Leung Lai and Hock Peng Chan, Statistics, Stanford University & C.V. Starr Professor, HKU, and Mathematics, NUS

Sequential Monte Carlo Methods for Computing Probabilities of Rare Events in Complex Models

It has been widely recognized that importance sampling provides a powerful tool for Monte Carlo evaluation of probabilities of rare events. This tool, however, has been hampered by difficulties in finding (or sampling from) the optimal proposal distribution and/or difficulties in

evaluating the importance weights in complex statistical models. We show how these difficulties can be circumvented by using recent advances in sequential importance sampling with re sampling. Applications to rare event simulation and percentile estimates in insurance mathematics and financial risk management, which involve serially dependent and high-dimensional data, are given.

Stephen Lee, Statistics and Actuarial Science, HKU

Variance Estimation for Sample Quantiles Using the m out of n Bootstrap

We consider the problem of estimating the variance of a sample quantile calculated from a random sample of size n . The r th-order kernel-smoothed bootstrap estimator is known to yield an impressively small relative error of order $O(n^{-r/(2r+1)})$. It nevertheless requires strong smoothness conditions on the underlying density function, and has a performance very sensitive to the precise choice of the bandwidth. The unsmoothed bootstrap has a poorer relative error of order $O(n^{-1/4})$, but works for less smooth density functions. We investigate a modified form of the bootstrap, known as the m out of n bootstrap, and show that it yields a relative error of order smaller than $O(n^{-1/4})$ under the same smoothness conditions required by the conventional unsmoothed bootstrap on the density function, provided that the bootstrap sample size m is of an appropriate order. The estimator permits exact, simulation-free, computation and has accuracy fairly insensitive to the precise choice of m . A simulation study is reported to provide empirical comparison of the various methods.

Ze-Hui Li, Department of Mathematics, Lan-Zhou University

Weak Convergence of a New Risk Model

An individual model in insurance risk is proposed describing more actually the real behavior of an insurance company by modeling the risk process directly based on the entrance process rather than merely using an approximate collective model. We study two cases where one policyholder is restricted to claim at most once or allowed to claim more times respectfully. Under the assumption that the entrance process follows an time-dependent Poisson process, we discuss the weak convergence of the risk processes in the last two cases. Interpreting the processes as cluster-marked shot noise processes by choosing suitable response functions and then utilizing a related result in the literature of shot noise process, we find conditions under

which asymptotic normality of the risk process (suitably normalized) holds. We also derive conditions under which the risk processes converge in distribution to non-normal $\alpha - stable$ variables.

Key words: Asymptotic normality, $\alpha - stable$ variables, cluster-marked shot noise process, heavy-tailed distribution, risk model, risk process, subexponential distribution.

Zhongfei Li, Center for Financial Engineering and Risk Management, Sun Yat-Sen University
Optimal Dynamic Portfolio Selection under Safety-First Rules

In this paper we study an optimal portfolio selection problem in a continuous-time financial market under safety-first criteria. In a Black-Scholes setting, we obtain closed-form explicit solutions of the best constant-rebalanced portfolios. Furthermore we analyze our results in relation to ones of the continuous-time mean-variance problem, and examine these results' implications and applications.

Zhengyan Lin, Department of Mathematics, Zhe-jiang University
Limiting Behavior of Strong Near-Epoch Dependent Random Variables

We introduce a new class of dependent sequences of random variables, which is a subclass of near-epoch dependent (NED) sequence, but can also be approximated by mixing sequences. we call them strong near-epoch dependent sequence. Many important econometric models, such as linear processes, a sort of popular nonlinear models defined by nonlinear difference equations, ARMA model, GARCH model etc., are strong NED under usual conditions.

Under dependence conditions substantially weaker than that for NED sequences, we show a p -order, $p > 2$ (maximum) moment inequality for strong NED sequences. Then, using this inequality, we derive a central limit theorem and a functional central limit theorem and based on these results, we can also obtain limit distributions of many important processes with strong NED innovations, such as linear processes with strong NED innovations. Moreover, we show a result on the variances of partial sums of a strong NED sequence, but it is usually considered as a prior-assumption in discussing the large sample behavior for an NED sequence.

Sai Wan Elias Shiu, Statistics and Actuarial Science, University of Iowa & Applied Mathematics, HKPU

The Time Value of Ruin in a Sparre Andersen Model

This paper considers a Sparre Andersen collective risk model in which the distribution of the interclaim time is that of a sum of n independent exponential random variables; thus the Erlang(n) model is a special case. The analysis is focused on the function $\phi(u)$, the expected discounted penalty at ruin, with u being the initial surplus. The penalty may depend on the deficit at ruin and possibly also on the surplus immediately before ruin. It is shown that the function $\phi(u)$ satisfies a certain integrodifferential equation and that this equation can be solved in terms of Laplace transforms, extending a result of X. Sheldon Lin. As a consequence, a closed form expression is obtained for the discounted joint probability density of the deficit at ruin and the surplus just before ruin, if the initial surplus is zero. For this formula and other results, the roots of Lundberg's equation in the right half of the complex plane play a central role. Also, it is shown that $\phi(u)$ satisfies Shuanming Li's Renewal Equation. Under the assumption that the penalty depends only on the deficit at ruin and that the individual claim amount density is a combination of exponential densities, a closed form expression for $\phi(u)$ is derived. In this context, known results of the Cauchy matrix are useful, which are explained in the Appendix. Surprisingly, certain results are best expressed in terms of divided differences, a topic deleted from the actuarial examinations at the end of last century.

Ken Siu, Department of Mathematics, City University of Hong Kong

On Pricing Derivatives under GARCH Models: A Dynamic Gerber-Shiu's Approach

This paper proposes a method for pricing derivatives under the GARCH assumption for underlying assets in the context of a "dynamic" version of Gerber-Shiu's option-pricing model. Instead of adopting the notion of local risk-neutral valuation relationship (LRNVR) introduced by Duan (1995), we employ the concept of conditional Esscher Transforms introduced by Bühlmann et al. (1996) to identify a martingale measure under the incomplete market setting. One advantage of our model is that it provides a unified and convenient approach to deal with different parametric models for the innovation of the GARCH stock-price process. Under the conditional normality assumption for the stock innovation, our pricing result is consistent with that of Duan (1995). In line with the celebrated Gerber-Shiu's option pricing model, we can justify the pricing result within the dynamic framework of utility maximization problems which makes the economic intuition of the pricing result more appealing. In fact, the use of the Esscher Transform for option valuation can also be justified by the minimization of the

relative entropy between the statistical probability and the risk-neutralized pricing probability. Numerical results for the comparison of our model with the Black-Scholes option pricing model are presented.

LiXin Song, Department of Applied Mathematics, Dalian University of Technology
Semiparametric Credibility Ratemaking Using a Piecewise Linear Prior

A practical estimate on the credibility formula is presented, where a piecewise linear function is taken as the approximation of the prior distribution and applied to the credibility theory. The convergence of the approximation is analyzed. Simulation results for the lognormal-lognormal mixture show the effectiveness of the proposed estimate on the credibility.

Chun Su, Department of Statistics & Finance, University of Science and Technology of China
On the Behavior of the Product of Independent Random Variables

It is well-known that the heavy-tailed distributions play a very important role in the insurance and finance business. There are many researches which discussed the problems under the assumptions of heavy-tailed distributions, although there are also some with light-tailed distributions.

On the other side, the products of independent random variables are widely used in the insurance and finance business to describe the effect of the interest, for example, the ARCH process and etc.

In the present, we study the tail behaves of the distributions of the products of independent random variables and discover that the the product operation obviously effect the tail behaves of the distributions. For example, for two independent non-negative random variables X and Y , the distribution of XY may be heavy-tailed, even though the distributions of both of X and Y are light-tailed, especially, when the distribution of X belongs to Class $\mathcal{L}(\gamma)(\gamma > 0)$ and is continuous, then the distribution of XY belongs to Class \mathcal{L} if and only if $P(Y > y) > 0, \forall y > 0$. This fact shows that the assumption of light-tailed distributions must be used with caution.

The aim of this work is to make clear the relationship between the tail behaves of the distributions of X or Y and the product distribution of XY . We discussed two subjects: one of them is that the distributions from light-tailed into heavy-tailed through multiplication; and the other one is that from some Classes of heavy-tailed distributions to the pristine Classes,

and give some results on these subjects.

Keywords: Class $\mathcal{L}(\gamma)$, Class \mathcal{L} , Class \mathcal{M} , Class \mathcal{M}^* , Class \mathcal{A}^* , Independent product, Semi- \mathcal{L} , Condition Γ

Duo Wang, Department of Financial Mathematics, Beijing University

Asset Pricing Dynamics with Time-Varying Second Moment

We develop a simple behavioral asset pricing model with fundamentalists and chartists to study price behavior in financial market. Within our model, the market impact of the weighting process of the conditional mean and variance of the chartists and investors' reactions are analysed. Price dynamics of the deterministic model under under/over-reactions are analyzed and some connections between the statistical properties of the nonlinear stochastic system (such as distribution density and autocorrelation coefficients of returns) and the stability and bifurcation of the underlying deterministic system are established. The model displays some of the stylised facts, such as fat tails, skewness, high kurtosis and long memory, observed in high frequency financial data.

Guojing Wang, Department of Mathematics, Suzhou University

Ruin Probabilities for a Risk Process with Stochastic Return on Investments

We consider a risk process with stochastic return on investments. The basic risk process is the classical risk process while the return on investments generating process is a compound Poisson process plus a Brownian motion with positive drift. We obtain an integral equation for the ultimate ruin probability which is twice continuously differentiable under certain conditions. We then derive explicit expression for the lower bound for the ruin probability. We also study a joint distribution related to exponential functionals of Brownian motion which is required in the explicit expressions for the lower bound.

Rongming Wang, Department of Statistics, East China Normal University

On Erlang(2) Risk Process Perturbed by Diffusion

In this paper, we consider an Erlang(2) risk process perturbed by diffusion. From the extreme value distribution of Brownian motion with drift and the renewal theory, we show that the survival probability satisfies an integral equation. We then give the bounds for the ruin probability and the ruin probability caused by claim. By introducing a random walk associated

with the proposed risk process, we define an adjustment-coefficient. The relation between the adjustment-coefficient and the bounds is given and the Lundberg-type inequality for the bounds is obtained.

Sam Wong, Information & Systems Management, HKUST

Efficient Implementation of Universal Portfolio by Sequential Importance Sampling

Universal portfolio is a sequential investment scheme which attains the wealth of the best constant rebalancing portfolio over a long investment period. Also, its rate of approaching that wealth is an increasing function of the number of stocks in the portfolio. However, the proposed implementation (Cover and Ordentlich, 1996) of the universal portfolio is a recursive algorithm which requires storage growing in the order of n^{m-1} where m is the number of stocks and n is the length of investment period. Its computational complexity restricts the theory to be applied only to very small portfolios. In this article, we derive a sequential importance sampling (SIS) algorithm from an explicit formula of the universal portfolio. The formula shows that the universal portfolio can be estimated efficiently by simulating partitions of $\{1, \dots, n\}$ from a probability distribution. Our algorithm significantly reduces the computational burden and enables the universal portfolio to be applied to a large number of stocks. Both simulated and real data examples will be used to illustrate the efficiency of the SIS implementation.

Zhigang Xie, School of Finance, Shanghai University of Finance and Economics

Risk Construction of Chinese Insurers: Identification of Methodologies

The China Insurance Regulatory Commission (CIRC) issued its Solvency Regulation based on both Required Solvency Margin and the Risk Indicators on 24th March 2003, while it set up a research project to follow up with the progress of the global framework for insurer solvency assessment and particularly with those models consistent to the three pillar principles of the New Basel Accord. The foundation of the research project focuses on the identification of risks faced by Chinese insurers. The paper is to analyse the methodologies for identifying insurer risks, and to reach a initial profile of risk construction for Chinese insurers.

It starts from the anatomy of insurance company and product process, summarises the typical classifications of risks and explores possible contributions by statistical and mathematical modeling, cause-effect and case study approaches for risk analysis.

It finally tries to propose a risk control approach for Chinese insurers under IAIS' corporate governance core principles and consistent to the three pillars principle of the new Basel Accord.

Hailiang Yang, Statistics and Actuarial Science, HKU

Ruin Theory: Martingale Approach

Gerber (1973) shows how martingale techniques can be used to derive results concerning the probabilities of ruin. In this talk, I will first briefly give a survey on the martingale method in ruin theory. The Lundberg inequality for ruin probability under a discrete time insurance risk model by using martingale method will be presented. The method will be extended to a model with stochastic interest income. The Lundberg type inequality for the distribution of severity of ruin, the distribution of surplus before ruin, and the joint distribution of surplus before and after ruin will also be presented.

Kam C. Yuen, Statistics and Actuarial Science, HKU

Some Ruin Problems for a Risk Process with Stochastic Interest

We consider a risk process with stochastic interest. We study the expected value of a discounted penalty function at ruin which is a function of the surplus immediately prior to ruin and the deficit at ruin. We derive an integro-differential equation for the expected value and its exact solution. We also examine a lower bound for the ruin probability of the risk process.

Key words: Integro-differential equation, lower bound, penalty function, ruin probability, stochastic interest.

Xinsheng Zhang, Department of Statistics, Fudan University

The Distribution of Exit Time for One Dimensional Diffusion Processes

We study how to calculate the exact distribution of exit time for one dimensional diffusion. Some sufficient conditions such that the distribution of exit time from finite or infinite interval can be expressed as the eigenfunction expansion are obtained. Some examples are also discussed.