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Trigonal algebraic surfaces and applications

Gonality of a complex projective curve C is defined as the minimal degree d of the finite covers of C over $\mathbb{P}^1$. This number d divides algebraic curves into some classes, $\mathbb{P}^1$, *hyperelliptic* ($d = 2$), *trigonal* ($d = 3$), and *d-gonal.* For example, $d = 1$ if $g(C) = 0$; $d = 2$ if $g = 1$ or 2; $d = 2$ or 3 if $g = 3$ or 4.

Similarly, the gonality d of an algebraic surface X can be defined as the minimal degree of generically finite covers of X over some *ruled surfaces* (not just $\mathbb{P}^2$). So algebraic surfaces are divided into classes: *ruled, hyperelliptic, trigonal*, and *d-gonal.* Because double cover is well understood now, we have a good method to classify hyperelliptic surfaces, which include surfaces with a genus 2 fibration, surfaces whose canonical maps are of degree 2, and surfaces whose invariants satisfy $c_1^2 < 3p_g - 7$.

We will talk about our triple cover method and its applications in the study of trigonal surfaces. Precisely, we will give upper bounds on the slope of trigonal fibrations, and describe by using rank two vector bundles the surfaces whose canonical maps are of degree 3 (joint work with Zhijie Chen). We will also present a cubic defining equation for each rational triple singularity of dimension two (joint work with Zhijie Chen, Rong Du and Fei Yu). Finally, we will show the relationship between rank two vector bundles on an algebraic surface and the discriminant curves of binary cubic forms.