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Analytic continuation of certain germs of holomorphic immersions between Fano manifolds of Picard number 1

Let X be a Fano manifold equipped with a choice of Chow component $\mathcal{K}$ of minimal rational curves. At a general point x of X the variety of minimalr rational tangents (VMRT) is the collection of projectivizations of vectors tangent to a minimal rational curve passing through x. We say that VMRTs are linear if at a general point x of X the VMRT is a union of linear subspaces (of the same dimension). This is in particular the case when the normal bundle of a general minimal rational curve is trivial.

In what follows we assume that X is of Picard number 1. With geometric applications in mind we consider the problem of analytic continuation of local holomorphic maps from X into a Fano manifold Y of Picard number 1. We say that X exhibits the Cartain-Fubini extension property if the following holds true. Let Y be any Fano manifold of Picard number 1 equipped with a Chow component of minimal rational curves, and $f : (X, x_0) \to (Y, y_0)$ be a germ of VMRT-preserving biholomorphic map, then f extends to a biholomorphism from X to Y. In a joint work with Jun-Muk Hwang, we established in 2001 that any Fano manifold X exhibits the Cartain-Fubini extension property provided that at a general point of X, the Gauss map is generically injective at the VMRT at x. In a more recent joint work in 2004, we generalized the method to show that the Cartan-Fubini extension property holds true provided that the VMRT at a general point is non-linear. As an application this shows that any generically finite surjective holomorphic mapping $g : X' \to X$ is rigid when the target manifold is fixed.

As far as the Cartan-Fubini extension property is concerned, it is natural to try to generalize to the non-equidimensional situation. In other words, we can consider the question of extension of a germ of holomorphic immersion $f : (X, x_0) \to (Y, y_0)$, where Y denotes a Fano manifold of Picard number 1 equipped with a Chow component of minimal rational curves, $x_0 \in X$ and $y_0 \in Y$ are general points, under the assumption that f transforms the VMRT $\mathcal{C}_x$ at a general point x of a neighborhood of x_0 into the VMRT $\mathcal{D}_{f(x)}$ at $f(x)$. Here we focus on the situation where the Gauss map at a general point of $\mathcal{C}_x$ is generically injective. Assume that a general point of $[df](\mathcal{C}_x) \subset \mathcal{D}_{f(x)}$ is smooth. We can then show that f extends to a meromorphic mapping under an assumption on the the Gauss map of $\mathcal{D}_{f(x)}$ at a general point of $[df](\mathcal{C}_x)$. This condition is nontrivial even when all VMRTs are nonsingular.

As an application we consider local holomorphic mappings between rational homogeneous manifolds of Picard number 1. This type of situation occurs in connection with rigidity problems on proper holomorphic mappings between bounded symmetric domains. It was proven by Tsai (1993) that when domain and target bounded symmetric domains are of the same rank ≥ 2, then any proper holomorphic mapping f is totally geodesic. A bounded symmetric domain can be realized as an open subset of its compact dual by means of the Borel embedding, and the properness and rank conditions imply that f sends VMRTs into VMRTs. Here we examine the general question whether such local holomorphic maps between rational homogeneous manifolds of Picard number 1 are necessarily totally geodesic.