Graduate Courses
(Updates Spring 2007)

MATH6203 Several Complex Variables

Professor Ngaiming Mok

Meeting Time and Venue: Every Wednesday, 3:00pm – 5:30pm, Room 517, Meng Wah Complex, starting 31 January 2007 – 2 May, 2007

In this introductory course we aim at familiarizing students with basic analytic and algebraic techniques in the study of functions of several complex variables

Domains in complex Euclidean spaces of dimension $n > 1$ may exhibit properties fundamentally different properties those of plane domains, starting with the phenomenon discovered by Hartogs that for certain domains holomorphic functions defined on them can automatically be analytically continued to strictly bigger domains. We will first of all use elementary analytic techniques such as the power series method and the Cauchy integral formula to study the Hartogs phenomenon and the related notions of domains of holomorphy and holomorphic convexity. Generalizing subharmonic functions we have in Several Complex Variables the notion of plurisubharmonic functions. Basic properties of such functions and some relations between plurisubharmonicity and the Hartogs phenomenon will be explained.

Domains of holomorphy are maximal domains for certain holomorphic functions. In terms of partial differential equation, it is possible to characterize domains of holomorphy by the solvability of Cauchy-Riemann equations. We will examine the case of a domain with smooth boundary, which is a domain of holomorphy if and only if defining functions of the boundary satisfy certain differential inequalities, leading to the notion of (strictly) pseudoconvex domains. We will explain in this context the solvability of the Cauchy-Riemann equation for (0,1) forms with L^2 -estimates due to Hörmander. Plurisubharmonic weight functions will play an important role in the estimates.

The local study of holomorphic functions in Several Complex Variables translates to the study of the algebra of germs of holomorphic functions at a given base point in n-dimensional complex Euclidean space, which is equivalently the algebra of convergent power series in n complex variables. On the geometric side (germs of) complex-analytic subvarieties are defined by zeros of ideals of (germs of) holomorphic functions, and the algebraic study of convergent power series lead to geometric properties on complex-analytic subvarieties. In this respect we will examine basic results such as the Weiestrass Preparation and Division Theorems and derive some elementary properties on germs of complex-analytic subvarities, e.g., in relation to their decomposition into irreducible components.

For the understanding of the course the student needs to have basic knowledge on functions of a complex variable, supplemented if necessary with some rudimentary knowledge on complex potential theory such as subharmonic functions. To understand the discussion on the Cauchy-Riemann equation with L^2-estimates it is desirable that the student has some background on Hilbert spaces, such as the Riesz reprentation theorem and the Hahn-Banach Theorem. On the algebraic side the student should be familiar with some basic notions in abstract algebra, for instance the notions of unique factorization domains and noetherian rings. The bibliography below is for reference only. The course will not follow closely any of the textbooks.

[FG] Fritzsche, Klaus and Grauert, Hans: *Several Complex Variables*, Springer-Verlag, Berlin-Heidelberg-New York, 1976.

[G] Gunning, Robert C.: *Introduction to Holomorphic Functions of Several Variables*, Volumes I and II, Brooks/Cole Publishing Company, Pacific Grove, California, 1990.

[H] Hörmander, Lars: *An introduction to complex analysis in several variables*, North-Holland, Amsterdam, 1990.

[N] Narasimhan, R.: *Several Complex Variables*, University of Chicago Press, Chicago, 1971.