THE UNIVERSITY OF HONG KONG

Institute of Mathematical Research
Department of Mathematics

COLLOQUIUM

Approximating power series solutions by polynomial solutions

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Abstract

Given a system of polynomial equations, $f(t, x) = 0$, in several variables x and in 1 parameter t, does there exist a function $x = x(t)$ which is a polynomial (or fraction of polynomials) in t and which solves the system, i.e. $f(t, x(t))$ equals 0? Such a solution is a "rational solution". Do there exist enough rational solutions to approximate every power series solution $x(t)$ to arbtirary order? The problem of answering the first question is Hilbert's 10th problem for $\mathbb{C}(t)$ which is expected to be negative -- there is no algorithm for determining whether or not a rational solution exists. For the second problem, the "Weak Approximation Problem", Hassett and Tschinkel conjecture a very simple answer: there are enough rational solutions precisely if, for a "typical" value of the parameter, $t = a$, the system $f(a, x) = 0$ is "rationally connected", i.e., for every pair of solutions x' and x'' there is a rational function $x = x(s)$ of one variables such that $f(a, x(s)) = 0$ and which connects x' and x'', i.e., $x(0) = x'$ and $x(1) = x''$.

I will discuss the topological and number theoretic motivation of this conjecture. I will discuss the evidence for the conjecture due to Hassett − Tschinkel, Hassett, Knecht, Xu and Colliot-Théléne − Gille. Then I will discuss a new approach of Mike Roth and myself putting this conjecture in the larger context of "algebro-geometric analogues of topological obstruction theory".

This will be a broad audience talk. No background in algebraic geometry will be necessary.

Date:	**April 6, 2009 (Monday)**
Time:	**4:15 – 5:15pm**
Place:	**Room 517, Meng Wah Complex, HKU**

Tea will be held in Room 516, Meng Wah Complex at 4:00pm

All are welcome