THE UNIVERSITY OF HONG KONG

Institute of Mathematical Research
Department of Mathematics

GEOMETRY SEMINAR

From the Bergman kernel to holomorphic isometries: a method of analytic continuation

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Abstract

Motivated by a problem from Arithmetic Geometry raised by Clozel-Ullmo, we study the question of characterizing germs of holomorphic isometric immersions between bounded domains with respect to the Bergman metric.

Extension and rigidity problems for holomorphic isometries into possibly infinite-dimensional space forms dated back to works of Bochner and Calabi. For a bounded domain $D \Subset \mathbb{C}^n$ equipped with the Bergman kernel $K(z,w)$, the function $\log K_D(z,z)$ serves as a potential function for the Bergman metric ds^2_D, and the choice of an orthonormal basis for the Hilbert space $H^2(D)$ of square-integrable holomorphic functions defines a holomorphic isometric embedding of (D, ds^2_D) into the infinite-dimensional projective space $\mathbb{P}^\infty$ equipped with the Fubini-Study metric. In the simply connected case interior extension results already follow from Calabi's seminal work in 1953 on the subject. Here we are primarily concerned with extension beyond the boundary for bounded domains with specific realizations, notably bounded symmetric domains in their Harish-Chandra realizations. The upshot is that the graph of a germ of holomorphic isometry extends algebraically in the latter case. On the other hand, we have found examples of proper holomorphic isometric embeddings of the Poincaré disk into bounded symmetric domains which are not totally geodesic, giving in particular counter-examples to a conjecture of Clozel-Ullmo's.

Bounded symmetric domains may be taken as prototypes, and generalizing the methods developed for these domains the problem of analytic continuation of holomorphic isometric immersions has now been solved in a very general setting. As an example, bounded symmetric domains share the common feature of admitting projective compactifications, given by the Borel embedding, such that the Bergman kernel $K(z,w)$ extends as a rational function in $(z, \overline{w})$ to the compactified space. We showed that the latter extension property is essentially what characterizes the Borel embedding as a 'canonical' embedding up to finite coverings ramified outside the domain, and the same applies whenever the Bergman kernel $K(z,w)$ is rational in $(z, \overline{w})$, in particular to bounded homogeneous domains realized as (unbounded) Siegel domains of the second kind.

Date:	**September 30, 2008 (Tuesday)**
Time:	**3:00 – 4:00pm**
Place:	**Room 517, Meng Wah Complex, HKU**

All are welcome