Abstracts

John C. Baez (University of California, Riverside and Centre for Quantum Technologies, Singapore)
Higher gauge theory, division algebras and superstrings
Classically, superstrings make sense when spacetime has dimension 3, 4, 6, or 10. It is no coincidence that these numbers are two more than 1, 2, 4, and 8, which are the dimensions of the normed division algebras: the real numbers, complex numbers, quaternions and octonions. We sketch an explanation of this already known fact and its relation to "higher gauge theory". Just as gauge theory describes the parallel transport of supersymmetric particles using Lie supergroups, higher gauge theory describes the parallel transport of superstrings using "Lie 2-supergroups". Recently John Huerta has shown that we can use normed division algebras to construct a Lie 2-supergroup extending the Poincaré supergroup when spacetime has dimension 3, 4, 6 and 10.

Peter Bouwknegt (Australian National University)
Leibniz algebroids and generalizations of geometry
In recent years there has been a flurry of interest in so-called 'generalized geometry' – as formalized by Hitchin and his students – motivated by its applications in String Theory. At an algebraic level this kind of generalized geometry arises from exact Courant algebroids. In this talk I will review some aspects of generalized geometry and discuss even more general geometries, including what are known in the physics literature as 'exceptional generalized geometries' arising from certain (non-exact) Courant algebroids, and Leibniz algebroids.

William D. Kirwin (Universität zu Köln and Instituto Superior Ténico, Lisboa)
Parallel transport and the Peter-Weyl theorem as the momentum space representation for a compact Lie group
Let K be a compact Lie group. As is well known, $L^2(K)$ can be interpreted as the "position-space" geometric quantization of K. In this talk, I will describe a "momentum-space" representation for K. I will also explain how this momentum-space representation is linked to the position-space representation via parallel transport with respect to a canonical connection in a certain Hilbert bundle. In particular, it is a result of Florentino-Matias-Mourão-Nunes that parallel transport along a particular geodesic from position-space to an intermediate fiber is exactly the generalized Segal-Bargmann(-Hall) transform. I will explain how their result can be extended to any other interior fiber (thus obtaining generalized Segal-Bargmann transforms), and moreover that when extended to momentum space, parallel transport yields the Peter-Weyl decomposition. This is joint work with S. Wu.

Wing Suet Li (Georgia Institute of Technology)
Intersection of subspaces and Horn inequalities for the eigenvalues of sums of selfadjoint operators
Let A, B, C be n by n selfadjoint matrices. It was conjectured by Horn in 1962 that the relation $A+B+C=0$ can be characterized by a set of inequalities (Horn inequalities) together with the trace equality. The conjecture was proved in the late 1990s due to the work of Klyachko, Knutson and Tao, using highly sophisticated tools from algebraic geometry. In this talk we will discuss some basic ideas on the intersection of subspaces that will lead to inequalities of the eigenvalues of sums of

selfadjoint operators, which leads to an elementary proof of the Horn inequalities that can be generalized to the finite Von Neumann albegras. I will also discuss some open questions that our techniques suggested.

Varghese Mathai (University of Adelaide)
Fractional index theory
In 1962, Atiyah and Singer defined the Dirac operator D, on any compact spin manifold M of even dimension. Recall that the analytic index of the Dirac operator is defined to be the integer $\text{index}(D) = \dim(\ker D) - \dim(\text{coker} D)$. Then the Atiyah-Singer index theorem for Dirac operators states that

$$\text{index}(D) = \int_M \widehat{A}(M).$$

Here the right hand side is the A-hat genus of the manifold M, which is a topological invariant. Since $\int_M \widehat{A}(M)$ continues to make sense for non-spin manifolds M (although however it may not be an integer in these cases), what corresponds to the analytic index in this situation, as the usual Dirac operator does not exist? My talk will be based on joint work with R.B. Melrose and I.M. Singer, in which we propose two solutions to the question above and relate them.

David A. Vogan (Massachusetts Institute of Technology)
Associated varieties of fundamental series representations
A basic problem in representation theory is to understand the restriction of a representation of a big group G to a subgroup K. I'll talk about the case when G is a real reductive Lie group and K is a maximal compact subgroup. To each irreducible representation X of G there is attached a complex algebraic variety $\text{AV}(X)$ (the *associated variety*) on which K acts. There are only finitely many possibilities for $\text{AV}(X)$; in the case of $\text{GL}(n, \mathbb{C})$, for example, $\text{AV}(X)$ must be the closure of a conjugacy class of nilpotent matrices.

What happens is that X restricted to K is approximately equal to the action of K on regular functions on $\text{AV}(X)$. Computing $\text{AV}(X)$ is therefore an approximate computation of X restricted to K.

I'll talk about some recent results of Ben Harris on the geometric nature of $\text{AV}(X)$, and illustrate with some computations of associated varieties for the split real form of E_8.

Yuanlong Xin (Fudan University and IMS, CUHK)
Some results on self-shrinkers
We consider the mean curvature flow for an isometric immersion of an n-dimensional manifold M into a Euclidean space, $X\colon M \to \mathbb{R}^{m+n}$. Namely, consider a one-parameter family $X_t = X(\cdot, t)$ of immersions $X_t\colon M \to \mathbb{R}^{m+n}$ with corresponding images $M_t = X_t(M)$ such that $\frac{d}{dt}X(x,t) = H(x,t)$ and $X(x,0) = X(x)$ $(x \in M)$, where $H(x,t)$ is the mean curvature vector of M_t at $X(x,t)$ in $\mathbb{R}^{m+n}$.

The simplest but important solutions to the above mean curvature flow equations are self-shrinkers which satisfy a system of quasi-linear elliptic PDE of the second order $H = -X^N/2$, where H is the mean curvature of the submanifold and X^N is the projection of X to the normal bundle of X.

In this talk I will present some results on volume growth, eigenvalue estimates for weighted Laplacian operator and compactness results for self-shrinkers.