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Fractional index theory

Abstract

In 1962, Atiyah and Singer defined the Dirac operator D, on any compact spin manifold M of even dimension. Recall that the analytic index of the Dirac operator is defined to be the integer $\mathrm{index}(D) = \dim(\ker D) - \dim(\mathrm{coker} D)$. Then the Atiyah-Singer index theorem for Dirac operators states that

$$\mathrm{index}(D) = \int_M \widehat{A}(M).$$

Here the right hand side is the A-hat genus of the manifold M, which is a topological invariant. Since $\int_M \widehat{A}(M)$ continues to make sense for non-spin manifolds M (although however it may not be an integer in these cases), what corresponds to the analytic index in this situation, as the usual Dirac operator does not exist? My talk will be based on joint work with R.B. Melrose and I.M. Singer, in which we propose two solutions to the question above and relate them.