# Workshop on Complex Geometry

# July 26-28, 2012

# Room 210, Run Run Shaw Bldg., HKU

# **Program and Abstracts**



Institute of Mathematical Research The University of Hong Kong

# Speakers:

Rong Du	East China Normal University	
Lawrence Ein	University of Illinois, Chicago, USA	
Pak-Tung Ho	Sogang University, Korea	
Xiaojun Huang	Rutgers University, USA	
Jun-Muk Hwang	KIAS, Korea	
Jaya Iyer	Institute of Mathematical Sciences, India	
Sui-Chung Ng	Temple University, USA	
Tuen-Wai Ng	The University of Hong Kong	
Mounir Nisse	Texas A&M University, USA	
Yum-Tong Siu	Harvard University, USA	
Shengli Tan	East China Normal University	
Wing-Keung To	National University of Singapore	
Mingxi Wang	Max-Planck Institut, Germany	
Sai-Kee Yeung	Purdue University, USA	

Organizers: Tuen-Wai Ng & Ngaiming Mok

Email: imr@maths.hku.hk

# Workshop on Complex Geometry

# July 26 - 28, 2012

# Room 210, Run Run Shaw Bldg., HKU

Time / Date	July 26 (Thur)	July 27 (Fri)	July 28 (Sat)
9:30 - 10:30	Siu	Ein	Huang
10:30 - 10:50	Tea Break		
10:50 - 11:50	То	lyer	Yuan
Lunch Break			
14:00 - 15:00	Yeung	Hwang	Ng TW
15:10 - 16:10	Ng SC	Tan	Wang
16:10 - 16:30	Tea Break		
16:30 - 17:30	Но	Du	Nisse

# PROGRAM



9:30 – 10:30 **Yum-Tong Siu**, Harvard University, USA *Techniques of multipliers in regularity problems of partial differential equations* 

Tea Break

10:50 – 11:50 **Wing-Keung To,** National University of Singapore *Volume of analytic subvarieties: some estimates and applications* 

Lunch Break

14:00 – 15:00Sai-Kee Yeung, Purdue University, USA<br/>On Kähler properties of mapping class groups

15:10 – 16:10 **Sui-Chung Ng**, Temple University, USA Holomorphic projective connection and splitting of sequences of holomorphic vector bundles

Tea Break

16:30 – 17:30 **Pak-Tung Ho**, Sogang University, Korea *Results related to Yamabe and CR Yamabe problem* 



9:30 – 10:30 **Lawrence Ein**, University of Illinois, Chicago, USA *Jacobian multiplier ideals* 

Tea Break

10:50 – 11:50 **Jaya Iyer**, Institute of Mathematical Sciences, India Stability of cotangent bundle on some moduli space of bundles and some variants

Lunch Break

14:00 – 15:00 **Jun-Muk Hwang**, KIAS, Korea Webs of Lagrangian tori in projective symplectic manifolds

15:10 – 16:10 **Shengli Tan**, East China Normal University Global and local Chern numbers of families of algebraic curves and applications

Tea Break

16:30 – 17:30 **Rong Du**, East China Normal University Numerical invariants of singularities and complex Plateau problem



9:30 – 10:30 **Xiaojun Huang**, Rutgers University, USA Local hull of holomorphy for a real submanifold in C<sup>n</sup>

Tea Break

10:50 – 11:50 **Yuan Yuan**, Johns Hopkins University, USA *Rigidity of the local holomorphic isometries between ball and products of balls* 

*Lunch Break* 

14:00 – 15:00 **Tuen-Wai Ng**, The University of Hong Kong Borcea's variance conjecture on the critical points of polynomials

15:10 – 16:10 **Mingxi Wang**, Max-Planck Institut, Germany Moduli space of Higgs bundles over real algebraic curves

Tea Break

15:10 – 16:10 **Mounir Nisse**, Texas A&M University, USA *The k-convexity of the amoeba complement and the generalized order map* 

## Rong Du, East China Normal University

Numerical invariants of singularities and complex Plateau problem

We introduce some invariants for singularities. These invariants measure in some sense how far the resolution manifolds of singularities are away from having global complex coordinates. As an application, we relate one of these invariants with a CR invariant to deal with complex Plateau problem.

Lawrence Ein, University of Illinois, Chicago, USA

### Jacobian multiplier ideals

We desribe joint work with Ishii and Mustata. We introduce Jacobian multiplier ideals for singular varieiteis and prove that they enjoy many nice properties as the usual multiplier ideals for smooth varieiteis.

**Pak-Tung Ho**, Sogang University, Korea Results related to Yamabe and CR Yamabe problem

In this talk, I will talk about the Yamabe problem and CR Yamabe problem. I will mention the results about Yamabe flow and CR Yamabe flow, and the uniqueness of the Yamabe problem and the CR Yamabe problem. I will also talk about the generalization of the Yamabe and CR Yamabe problem, namely, the problem of prescribing scalar curvature and Webster scalar.

### Xiaojun Huang, Rutgers University, USA

Local hull of holomorphy for a real submanifold in  $C^n$ 

For a subset in a complex space, its local hull of holomorphy is defined as the intersection of all pseudo convex domains containing the set. In this talk we describe the holomorphic hull of a class of submanifolds in a complex space.

### Jun-Muk Hwang, KIAS, Korea

Webs of Lagrangian tori in projective symplectic manifolds

Given a Lagrangian torus A in a projective irreducible symplectic manifold M, Beauville raised the question whether some birational model of M admits a fibration with A as a fiber. In a joint work with Richard Weiss, we give an affirmative answer. Our work employs two different tools: the theory of action-angle variables for algebraically completely integrable Hamiltonian systems and Wielandt's

theory of subnormal subgroups. To apply these tools, we will introduce the notion of webs of submanfields and study their monodromy groups.

#### Jaya Iyer, Institute of Mathematical Sciences, India

Stability of cotangent bundle on some moduli space of bundles and some variants

In this talk we will discuss stability of cotangent bundle on certain moduli space of bundles and also discuss logarithmic version for some examples.

#### Sui-Chung Ng, Temple University, USA

Holomorphic projective connection and splitting of sequences of holomorphic vector bundles

Let  $0 \to A \to C \to B \to 0$  be an exact sequence of holomorphic vector bundles over a complex manifold X. We say that the sequence splits holomorphically if C is holomorphically isomorphic to the direct sum of A and B. For a complex manifold M, we have on every complex submanifold  $S \subset M$ a natural sequence of vector bundles (the tangent sequence) by pulling-back the tangent bundle of M.

In this talk, we will look at complex manifolds admitting holomorphic projective connections and their complex submanifolds with splitting tangent sequences. We will discuss how these submanifolds inherit holomorphic projective connections from their ambient manifolds. Possible generalizations to holomorphic conic connections will also be mentioned if time permits.

### Tuen-Wai Ng, The University of Hong Kong

Borcea's variance conjecture on the critical points of polynomials

One of the main open problems in geometry of polynomials is the Sendov's conjecture on the location of critical points of a polynomial of one complex variable. Recently, J.Borcea has suggested a new approach to Sendov's conjecture through his variance conjecture. In this talk we will give an introduction to Borcea's variance conjecture.

#### Mounir Nisse, Texas A&M University, USA

The k-convexity of the amoeba complement and the generalized order map

Amoebas are the images under the logarithmic map of algebraic (or analytic) varieties of the complex algebraic torus. They inherit some algebraic, geometric, and topological properties of the variety itself. A global statement generalizing convexity of amoeba complement components was found by André Henriques, and he proved a weaker version of this property for the complement of amoebas. We show a stronger version which complete the generalization of the k-convexity of the amoeba complement in higher codimension. Moreover, we define the generalized order maping in higher codimension, which is already well defined for hypersurface by Passare and Tsikh. (This is a joint work with Frank Sottile.)

# Yum-Tong Siu, Harvard University, USA

Techniques of multipliers in regularity problems of partial differential equations

Will discuss the history, known results, as well as current and possible future developments of the techniques of multipliers in regularity problems of partial differential equations.

## Shengli Tan, East China Normal University

Global and local Chern numbers of families of algebraic curves and applications

We will talk about some problems on the global and local Chern numbers of families of algebraic curves. As applications, we will construct families of hyperelliptic curves with the highest slopes, classify some families of curves over  $\mathbb{P}^1$  with 3 singular fibers and present optimal upper bounds on the abelian automorphism groups of surfaces of general type. This is a survey of some recent joint work with C. Gong, J. Lu, X. Lv, X.-L. Liu, F. Yu, K. Zuo.

# Wing-Keung To, National University of Singapore

Volume of analytic subvarieties: some estimates and applications

In this talk, I will discuss some joint works with J.-M. Hwang on bounding from below the volume of complex analytic subvarieties in certain Hermitian locally symmetric manifolds of non-compact type, as well as some applications related to Seshadri numbers, gonality of Riemann surfaces, and syzygies of pluricanonical systems. Some recent work in progress will also be discussed.

Mingxi Wang, Max-Planck Institut, Germany

### Moduli space of Higgs bundles over real algebraic curves

We study several involutions on the moduli space of Higgs bundles over a real curve: one from the real structure of non-abelian Betti moduli space, one from the real structure of non-abelian Dolbeault moduli space, and one a composition of the previous two. When the real structure of the curve is attached to a Schottky uniformization, there is a correspondence between representations of Schottky group and part of fixed points of our third involution. This correspondence settles a fundamental case of a problem of Faltings.

# Sai-Kee Yeung, Purdue University, USA

### On Kähler properties of mapping class groups

A group is called a Kähler group if it is isomorphic to the fundamental group of a projective algebraic manifold. A mapping class group can be considered as the orbifold fundamental group of a moduli space of curves, which is non-compact. It has been an open problem whether a mapping class group of genus at least 3 is a Kähler group of not. The purpose of the talk is to address the problem.

## Yuan Yuan, Johns Hopkins University, USA

Rigidity of the local holomorphic isometries between ball and products of balls

I will talk about a joint work with Yuan Zhang on local holomorphic isometric embeddings from the unit ball into the product of unit balls with respect to the normalized Bergman metrics up to conformal factors. Assume that each conformal factor is smooth Nash algebraic. Then each component of the map is a multi-valued holomorphic map between complex Euclidean spaces by the algebraic extension theorem derived along the lines of Mok and Mok-Ng. Applying holomorphic continuation and analyzing real analytic subvarieties carefully, each component is either a constant map or a proper holomorphic map between balls. The total geodesy of non-constant components then follows from a linearity criterion of Huang.