THE UNIVERSITY OF HONG KONG

Institute of Mathematical Research
Department of Mathematics

LECTURE SERIES

on

Some Aspects of the Conjecture of André-Oort

June 13, 2012 (Wednesday)
Room 210, Run Run Shaw Bldg., HKU

Mr. Lars Kühne
Departement Mathematik, ETH Zentrum, Switzerland

An effective result of André-Oort type

3:00 – 4:00pm

Abstract

The André-Oort Conjecture (AOC) states that the irreducible components of the Zariski closure of a set of special points in a Shimura variety are special subvarieties. Here, a special variety means an irreducible component of the image of a sub-Shimura variety by a Hecke correspondence. The AOC is an analogue of the classical Manin-Mumford conjecture on the distribution of torsion points in abelian varieties.

I will present a rarely known approach to the AOC that goes back to Yves André himself: Before the model-theoretic proofs of the AOC in certain cases by the Pila-Wilkie-Zannier approach, André presented the first non-trivial proof of the AOC in case of a product of two modular curves. In my talk, I discuss a result in the style of André's method, allowing to actually compute all special points in a non-special curve of a product of two modular curves.

4:00 – 4:20pm *Tea Break*

Professor Ngaiming Mok
The University of Hong Kong

On a geometric analogue of the André-Oort Conjecture for the Zariski closure of an infinite family of totally geodesic subvarieties of positive dimension

4:20 – 5:20pm

Abstract

Let Ω be a bounded symmetric domain, $\Gamma \subset \mathrm{Aut}(\Omega)$ be a torsion-free lattice, $X := \Omega/\Gamma$. Let $Z \subset X$ be an irreducible quasi-projective variety such that Z is the Zariski closure of the union of an infinite set of totally geodesic subvarieties $S_i \subsetneq Z$ of positive dimension. A geometric analogue of the André-Oort Conjecture is to ask whether Z admits a special structure. Under certain non-degeneracy condition one expects that Z also to be totally geodesic so that Z is also uniformized by a bounded symmetric domain.

We explain first of all how this can be established in the special case of the complex unit ball. In this case Z, of dimension $s+1 \geq 2$ is proven to be totally geodesic without any additional hypothesis. The idea is to generate an s-dimensional holomorphic family $\mathcal{F}$ of totally geodesic holomorphic curves on the universal covering ball $\mathbb{B}^n$, $s > 0$, so that the $s+1$-dimensional set Σ swept out by $\mathcal{F}$ can be extended to cross the boundary $\partial\mathbb{B}^n$, and properties of Z are derived from the asymptotic behavior of Σ as points approach $\partial\mathbb{B}^n$. A strengthening of the argument solves the problem in special cases such as the case where Ω is any bounded symmetric domain and Z is a complex surface.

All are welcome (especially graduate students)