THE UNIVERSITY OF HONG KONG

Institute of Mathematical Research
Department of Mathematics

WORKSHOP ON FUNCTION THEORY

Monday, 6 August, 2012

Room 210, Run Run Shaw Building, HKU

Morning Session

10:30 – 10:35	Ngaiming Mok (Director of IMR, The University of Hong Kong) *Opening remarks*
10:35 – 11:20	Mourad Ismail (University of Central Florida and King Saud University, USA) *R. Willam Gosper and his identities*
Coffee / tea break	
11:35 – 12:20	Sergei Kalmykov (Institute for Applied Mathematics of Far Eastern Branch of Russian Academy of Sciences, Russia) *Majorization principles and inequalities for polynomials and rational functions*
Lunch Break	

Afternoon Session

14:00 – 14:45	Elena Prilepkina (Institute for Applied Mathematics of Far Eastern Branch of Russian Academy of Sciences, Russia) *Generalized Stieljes transform and hypergeometric functions*
14:45 – 15:30	Chiu Yin Tsang (Hong Kong University of Science and Technology, Hong Kong) *Finite Blaschke products that share a set*
Coffee / tea break	
15:45 – 16:30	Kwok Kin Wong (The University of Hong Kong, Hong Kong) *Exact meromorphic stationary solutions of the real cubic Swift-Hohenberg equation*
Workshop dinner	

Organizer: Tuen Wai Ng

Abstracts

Mourad Ismail, University of Central Florida and King Saud University, USA

R. Willam Gosper and his identities

I will say something about Bill Gosper and his work. I will present some of the identities he discovered and where they come from and the mathematical concepts behind them. The left-hand sides of some sample formulas are:

$$\sum_{n=1}^{\infty} \frac{1}{n^2} \cos\left(\frac{\pi}{n+\sqrt{n^2+1}}\right) = ??$$

and

$$1 + \sum_{n=1}^{\infty} (x^{1/2} - c)(x^{1/4} - c) \cdots (x^{2^{-n}} - c) = ??$$

for $|1 - c| < 1,\ x \notin (-\infty, 0)$. If interested try to fill in the ?? in the right-hand sides.
The second identity is not very difficult to prove but the first is harder.

Sergei Kalmykov, Institute for Applied Mathematics of Far Eastern Branch of Russian Academy of Sciences, Russia

Majorization principles and inequalities for polynomials and rational functions

In my report new majorization principles for meromorphic functions will be presented. Also we will consider some applications of them, such as covering theorems for polynomials, Bernstein-type inequalities, coefficient estimates. Extremal cases will be presented in terms of n-fold complete coverings.

Elena Prilepkina, Institute for Applied Mathematics of Far Eastern Branch of Russian Academy of Sciences, Russia

Generalized Stieljes transform and hypergeometric functions

In this report we apply generalized Stieltjes transform representation to study the generalized hypergeometric function. Among the results thus proved are new integral representations, inequalities and the properties of the generalized hypergeometric function as a conformal map.

Chiu Yin Tsang, Hong Kong University of Science and Technology, Hong Kong

Finite Blaschke products that share a set

In this talk, I will give a solution of the following problem: under what conditions on two finite Blaschke products B_1, B_2 and two connected compact sets E_1 and E_2 of the open unit disk $\mathbb{D}$ of positive (hyperbolic) capacity do the preimages $B_1^{-1}(E_1)$ and $B_2^{-1}(E_2)$ coincide. It can be seen that the problem is closely related to the functional equation $C \circ B_1 = D \circ B_2$, where C, D are finite Blaschke products, by the application of potential theory (hyperbolic version). Then the solution can be given by Ritt's result on the factorization of finite Blaschke products (in the sense of compositions).

Kwok Kin Wong, The University of Hong Kong, Hong Kong

Exact meromorphic stationary solutions of the real cubic Swift-Hohenberg equation

The real cubic Swift-Hohenberg equation (RCSH)

$$\frac{\partial u}{\partial t} = \varepsilon u - \left(1 + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 u - u^3, \qquad \varepsilon \in \mathbb{R}. \tag{1}$$

is first proposed in the study of Rayleigh-Bénard convection in hydrodynamics. Since then the equation and its generalizations have been applied to a variety of physics and engineering problems, for example, laser and nonlinear optics. RCSH admits the stationary reduction

$$U'''' + aU'' + U^3 - U = 0, \; ' := \frac{d}{dZ}, \tag{2}$$

which is particularly important in the theory of pattern formation.
In this talk, we shall apply the Nevanlinna theory to show that all meromorphic solutions of (2) is either elliptic or elliptic degenerates. Then by using the subequation method of Conte and Musette, we shall obtain all these meromorphic soluions in explicit forms. One of these solutions appears to be new.