# **Conference on Complex Geometry**

June 25 - 28, 2013

## Room 210, Run Run Shaw Bldg., HKU

**Program and Abstracts** 



Institute of Mathematical Research Department of Mathematics

## Speakers:

Der-Chen Chang	Georgetown U., USA		
Yik-Man Chiang	HKUST, Hong Kong		
Lawrence Ein	U. Illinois, Chicago, USA		
Baohua Fu	Chinese Academy of Sciences, Beijing		
Dano Kim	Seoul National U., Korea		
László Lempert	Purdue U., USA		
Ngaiming Mok	HKU, Hong Kong		
Sui-Chung Ng	Temple U., USA		
Tuen-Wai Ng	HKU, Hong Kong		
Mihai Paun	KIAS, Korea		
Min Ru	U. Houston, USA		
Yum-Tong Siu	Harvard U., USA		
Xiaotao Sun	Chinese Academy of Sciences, Beijing		
Shengli Tan	East China Normal U., Shanghai		
Mina Teicher	Bar-Ilan U., Israel		
Wing-Keung To	National U. Singapore, Singapore		
Sai-Kee Yeung	Purdue U., USA		
Wanke Yin	U. Rouen, France		
Xiangyu Zhou	Chinese Academy of Sciences, Beijing		

Organizers: Tuen-Wai Ng, Ngaiming Mok & Xiaotao Sun

Email: imr@maths.hku.hk

# Conference on Complex Geometry

## June 25 - 28, 2013

# Room 210, Run Run Shaw Bldg., HKU

Time / Date	June 25 (Tue)	June 26 (Wed)	June 27 (Thur)	June 28 (Fri)	
9:30 - 10:30	Siu	Ein	Teicher	Yeung	
10:30 – 10:50 Tea Break					
10:50 - 11:50	Paun	Fu	Tan	Sun	
Lunch Break					
14:00 - 15:00	Lempert	Zhou	Ru	Mok	
15:10 - 16:10	Chang	NgTW	Chiang	То	
16:10 - 16:30	Tea Break				
16:30 - 17:30	Kim	Yin	NgSC		

## PROGRAM

## June 25, 2013 Tuesday

9:30 – 10:30 **Yum-Tong Siu**, Harvard U., USA Effective very ampleness

Tea Break

10:50 – 11:50 **Mihai Paun**, KIAS, Korea *Metrics with conic singularities and generic semi-positivity* 

Lunch Break

14:00 – 15:00 **László Lempert**, Purdue U., USA *Quantizing a Riemannian manifold* 

15:10 – 16:10 **Der-Chen Chang**, Georgetown U., USA *Heat kernels for a family of subelliptic operators* 

Tea Break

16:30 – 17:30 **Dano Kim**, Seoul National U., Korea *L*<sup>2</sup> *estimates and analytic adjoint ideals* 



9:30 – 10:30 **Lawrence Ein**, U. Illinois, Chicago, USA *Stability of syzygy bundles* 

Tea Break

10:50 – 11:50 **Baohua Fu**, Chinese Academy of Sciences, Beijing *Sixty years of compactifications of C<sup>n</sup>* 

Lunch Break

14:00 - 15:00Xiangyu Zhou, Chinese Academy of Sciences, Beijing<br/>Some recent results on L2 extension problem

15:10 – 16:10 **Tuen-Wai Ng**, HKU, Hong Kong On finite Blaschke products sharing preimage of connected compact sets

Tea Break

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16:30 – 17:30 **Wanke Yin**, U. Rouen, France *Convergence of formal equivalences between real analytic hypersurfaces in* C<sup>2</sup>



9:30 – 10:30 **Mina Teicher**, Bar-Ilan U., Israel Line arrangements – topology, combinatorics, and applications

Tea Break

10:50 – 11:50 **Shengli Tan**, East China Normal U., Shanghai *Jung's method of resolution and the invariants of a surface singularity* 

Lunch Break

14:00 – 15:00 **Min Ru**, U. Houston, USA On *M*-large divisors and their geometric and arithmetic properties

15:10 – 16:10 **Yik-Man Chiang**, HKUST, Hong Kong On eigen-solutions of Heun equation, affine Weyl group and Painlevé equation

*Tea Break* 

16:30 – 17:30 **Sui-Chung Ng**, Temple U., USA Holomorphic double fibration and the mapping problems of classical domains 9:30 – 10:30 **Sai-Kee Yeung,** Purdue U., USA *Examples for the transcendence of the derivative of a normalized uniformizing map* 

Tea Break

10:50 - 11:50Xiaotao Sun, Chinese Academy of Sciences, Beijing<br/>Etale fundmental groups and D-modules in characteristic p > 0

Lunch Break

14:00 – 15:00 **Ngaiming Mok**, HKU, Hong Kong Holomorphic isometries of the complex unit ball into bounded symmetric domains

15:10 – 16:10 **Wing-Keung To**, National U. Singapore *Hyperbolicity and families of polarized manifolds* 

#### Der-Chen Chang, Georgetown U., USA

Heat kernels for a family of subelliptic operators

We construct the heat kernel for the second-order operator  $\Delta_X = \frac{1}{2} \sum_{k=1}^n \left(\frac{\partial}{\partial x_k}\right)^2 + \frac{1}{2} \sum_{k=1}^n \left(x_k^2 \frac{\partial}{\partial y_k}\right)^2$ , which is a degenerate elliptic operator. Obviously, this operator is closely related to the Grushin operator  $L_G = \frac{1}{2} \left(\frac{\partial}{\partial x}\right)^2 + \frac{1}{2} \left(x^2 \frac{\partial}{\partial y}\right)^2$ . In this talk, we first study the geometry induced by the operator  $L_G$ . Given any two points in the space, the number of geodesics and the lengths of the geodesics are calculated. Then we find modified complex action functions and show that the critical values of this function will recover the lengths of the corresponding geodesics. We also find the volume element by solving a generalized transport equation. Finally, the formula for the heat kernel of the diffusion operator  $\frac{\partial}{\partial t} - \Delta_X$  is obtained. We also discuss asymptotic behavior of these kernels.

#### Yik-Man Chiang, HKUST, Hong Kong

On eigen-solutions of Heun equation, affine Weyl group and Painlevé equation

We derive an infinite expansion of biconfluent Heun equation, and show how one can recover eigensolutions of the differential equation from an affine Weyl group that is associated with a Painlevé equation.

Lawrence Ein, U. Illinois, Chicago, USA

Stability of syzygy bundles

We'll discuss joint work with R. Lazarsfeld and Y. Mustopa. We'll discuss results of the form if L is a sufficiently very ample line bundle on a smooth projective variety, then  $M_L$ , the first syzygy bundle of L is stable.

#### Baohua Fu, Chinese Academy of Sciences, Beijing

Sixty years of compactifications of  $C^n$ 

In 1954, Hirzebruch posed the problem to classify analytic compactifications of  $C^n$  of  $b_2 = 1$ . I shall give a survey on results obtained for this problem over the sixty years and then discuss the new direction initiated by Hassett-Tschinkel in 1999 on equivariant compactifications of  $C^n$ . This is based on joint works with Jun-Muk Hwang.

#### Dano Kim, Seoul National U., Korea

#### $L^2$ estimates and analytic adjoint ideals

We will discuss recent work of Blocki on  $L^2$  estimates with 'Poincaré type' weights and its application toward the coherence of analytic adjoint ideal sheaves of plurisubharmonic functions.

#### László Lempert, Purdue U., USA

#### Quantizing a Riemannian manifold

Typically, the first step in the quantization of a physical system is finding a Hilbert space whose vectors represent the quantum states of the system. Assuming we understand the classical configuration space, a Riemannian manifold M, geometric quantization provides a way to construct this Hilbert space. The Kähler version of geometric quantization constructs the quantum Hilbert space as the space of square integrable holomorphic sections of a certain line bundle over the tangent bundle TM, which is often the same thing as holomorphic  $L^2$  functions on TM. For this to be meaningful, one needs to choose a complex structure on TM and a weight function (because  $L^2$  refers to a weighted  $L^2$  space).

The talk will discuss my joint results with Szőke on how one can make these choices and whether the quantum Hilbert spaces corresponding to different choices are canonically isomorphic.

#### Ngaiming Mok, HKU, Hong Kong

Holomorphic isometries of the complex unit ball into bounded symmetric domains

In earlier works we proved that any germ of holomorphic isometry  $f: D \to D'$  (up to normalizing constants) with respect to the Bergman metric between two bounded domains extends to a proper holomorphic isometry whenever the Bergman metrics are complete, and that moreover the graph of the map extends to an affine-algebraic variety provided that the Bergman kernels are rational functions. In particular this applies to bounded symmetric domains in their Harish-Chandra realizations. We have also constructed examples of nonstandard holomorphic isometries of the Poincaré disk into certain bounded symmetric domains. It has been unknown for some time whether the complex unit ball  $B^n, n \geq 2$ , can be holomorphically and isometrically embedded in a nonstandard way into some bounded symmetric domains.

We give a construction of such isometric embeddings for  $n \ge 2$ , and show some examples where one can prove uniqueness up to automorphisms of the domain and target spaces. We also explain the relevance of holomorphic isometric embeddings of the complex unit ball for the Hyperbolic Ax-Lindemann Conjecture for arbitrary lattices in functional transcendence theory.

#### Sui-Chung Ng, Temple U., USA

#### Holomorphic double fibration and the mapping problems of classical domains

The rigidity or classification of special mappings among domains on complex manifolds is a traditional subject in Several Complex Variables. The proper holomorphic mappings between complex unit balls are among those who have attracted most attention and there have been various inputs from Algebraic Geometry, CR Geometry, PDE, etc. In this talk, we are going to look at the case of higher-rank Type-I irreducible bounded symmetric domains which remains rather unexplored comparing to the case of unit balls. We will try to illustrate some interesting linkages between the mapping problems of Type-I domains and those of generalized balls. This is by considering a very simple and natural holomorphic double fibration structure on certain flag varieties.

#### Tuen-Wai Ng, HKU, Hong Kong

#### On finite Blaschke products sharing preimage of connected compact sets

In this talk, we shall consider the problem of characterizing those finite Blaschke products sharing preimage of connected compact subsets with positive hyperbolic capacity of the standard unit disk. We shall see that this problem is closely related to the factorization of finite Blaschke products (in the sense of compositions) and also the problem of characterizing polynomials sharing the same Julia set. To solve this problem, we shall follow T.C. Dinh's approach to a similar problem for polynomials. This is a joint work with C.Y. Tsang.

#### Mihai Paun, KIAS, Korea

Metrics with conic singularities and generic semi-positivity

We will first present a few facts concerning the construction of metrics with conic singularities associated to pairs. We show next that these objects can be used in the investigation of the generic positivity properties of quotients of the cotangent bundles of projective manifolds.

#### Min Ru, U. Houston, USA

#### On M-large divisors and their geometric and arithmetic properties

Motivated by the classical Theorems of Picard and Siegel and their generalizations, we define the notion of an (essentially) M-large divisor for M > 0 (after Aaron Levin, Heier-Ru) and derive a "master" quantitative-type theorem (in the spirit of Nevanlinna-Schmidt). As consequences of this "master" theorem, we recover the recent results obtained in this direction (by Corvaja-Zannier, Evertse-Ferretti, Aaron Levin, and Ru, etc.), as well as derive some new results. The master theorem unifies the statements as well as the proofs in this direction.

Yum-Tong Siu, Harvard U., USA

#### Effective very ampleness

Will discuss the proof of the very ampleness part of the Fujita conjecture concerning the very ampless of mL + K for an effective m when L is an ample line bundle on a compact complex manifold of complex dimension n with canonical line bundle K.

#### Xiaotao Sun, Chinese Academy of Sciences, Beijing

Etale fundmental groups and D-modules in characteristic p > 0

This is a joint work with H. Esnault. For a smooth projective variety over an algebraically closed field of characteristic p > 0, we proved: (1) All the irreducible *D*-modules have rank 1 if and only if the commutator of the etale fundamental group is a pro-p-group. (2) Every *D*-module is a direct sum of rank 1 *D*-modules if and only if the etale fundamental group is abelian with no non-trivial *p*-power quotient. The above theorem with a result of Esnault-Mehta (Invent. math. 181 (2010), 449-465) together proves completely a conjecture of D. Gieseker (D. Gieseker: Ann. Sc. Norm. Super. Pisa, 4 (1975), 1-31).

#### Shengli Tan, East China Normal U., Shanghai

Jung's method of resolution and the invariants of a surface singularity

Jung's resolution of a surface singularity is a classical and powerful method, especially when we know explicitly the defining equations of the singularity. We are interested in getting a sequence of analytic and topological invariants form Jung's resolution, and applying them to the classification of surface singularities. This is a joint work with Jun Lu.

#### Mina Teicher, Bar-Ilan U., Israel

Line arrangements - topology, combinatorics, and applications

I shall describe topological, combinatorical properties of line arrangements and show connections between the topology and combinatorics. I shall also present applications to Algebraic Geometry (topology and classifications of algebraic surfaces).

#### Wing-Keung To, National U. Singapore, Singapore

Hyperbolicity and families of polarized manifolds

In this talk, I will discuss some joint works with Sai-Kee Yeung on hyperbolicity properties of the parameter spaces of certain families of polarized manifolds.

#### Sai-Kee Yeung, Purdue U., USA

Examples for the transcendence of the derivative of a normalized uniformizing map

The purpose of the talk is to introduce a problem of Lang on the transcendence of the derivative of a normalized uniformizing map for a Riemann surface, and to explain some examples, both in dimension one and some higher dimensional analogues.

#### Wanke Yin, U. Rouen, France

Convergence of formal equivalences between real analytic hypersurfaces in  $\mathbb{C}^2$ 

Let (M, p) and  $(M^*, p^*)$  be two real analytic hypersurfaces in  $\mathbb{C}^2$ . In this talk, we will talk about when a formally biholomorphic mapping sending (M, p) to  $(M^*, p^*)$  must be holomorphic. In particular, when n = 2, we will give an affirmative answer to this problem if (M, p) is not formally equvilent to the standard Levi-flat hyperpane (which is a necessary condition). This is a joint work with Nordine Mir.

#### Xiangyu Zhou, Chinese Academy of Sciences, Beijing

Some recent results on  $L^2$  extension problem

In this talk, we'll talk about some recent results about  $L^2$  extension problem with optimal estimate and applications.