THE UNIVERSITY OF HONG KONG

Institute of Mathematical Research
Department of Mathematics

MINI COURSE

Zagier's adele

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Abstract

Several years ago, Don Zagier, in the framework of the theory of mock modular forms, described a natural way to associate a real number and a sequence of p-adic numbers, one for every prime p, to a cusp Hecke eigenform. He claimed that this series constitute a rational adele class. We recal Zagier's construction. In the case of a weight two cusp Hecke eigenform, we discuss how a version of the Hodge decomposition for the formal group law allows one to prove Zagier's claim.

Lecture 1: Modular forms and weak harmonic Maass forms
June 20, 2014 (Friday), 4:00 – 5:00pm

Starting from a definition of modular forms, we introduce weak harmonic Maass forms and mock modular forms as their meromorphic parts. We recall several important facts from the theory of mock modular forms. In particular, we indicate a p-adic relation between a mock modular form and its shadow. That will allow us to explain Zagier's construction of an adele class (over rationals) associated in a natural way with a cusp Hecke eigenform.

Lecture 2: Modular parametrization of an elliptic curve and weak harmonic Maass forms
June 23, 2014 (Monday), 4:00 – 5:00pm

We recall and discuss a theorem of Eichler and Shimura which allows one to associate an elliptic curve with a weight two rational cusp Hecke eigenform. We further recall an observation by Eisenstein from the classical theory of elliptic functions which allows one to interpret weight zero weak harmonic Maass forms as a lifting of Weierstrass zeta-function. We also provide a contemporary interpretation of this observation in terms of a Hodge decomposition.

Lecture 3: Formal groups, their cohomology and Zagier's adele
June 25, 2014 (Wednesday), 4:00 – 5:00pm

We recall basic facts on one-dimensional commutative formal groups and an associated cohomology theory. We make use of results and constructions described in the previous talk in order to reformulate Zagier's adele claim in terms of the cohomology of a certain formal group, and describe a way to prove the claim in this framework.

Room 210, Run Run Shaw Bldg., HKU

All are welcome