Abstract

The broad theme of these lectures is the interplay between moduli spaces of geometric structures on surfaces and the representation theory of finite-dimensional algebras. One aspect of this is that certain canonical, geometrically-defined functions on moduli spaces of local systems turn out to coincide with generating functions arising in the representation theory of an associated quiver with relations. We discuss two complementary points of view on this relationship, those of 4d $N = 2$ supersymmetric field theory and of cluster algebras. We illustrate our larger theme through a detailed study of a basic but fundamental example, that of pure $N = 2$ Yang-Mills theory, where some of these canonical functions admit yet another interpretation as the Hamiltonians of a well-known integrable system, the relativistic Toda lattice.