THE UNIVERSITY  OF HONG KONG

Institute of Mathematical Research
Department of Mathematics

International School on Geometry, Groupoids and Quantization

2 - 5 November, 2013
Room 210, Run Run Shaw Building, HKU

Saturday, 2 November

Ngaiming Mok (Director, Institute of Mathematical Research, HKU)
14:00 - 14:05 *Opening remarks*

José M. Mourão (Instituto Superior Técnico, Lisboa)
Decomplexification of integrable systems, quantization and Kähler geometry (3 lectures)

14:05 – 14:55 Lecture 1: *Quantization in Kähler and real polarizations*

Coffee / Tea Break

15:10 – 16:00 Lecture 2: *Imaginary time flow of Kähler structures and decomplexification*

Coffee / Tea Break

16:30 – 17:20 Lecture 3: *Imaginary time flow of holomorphic sections and tropicalization of divisors*

Sunday, 3 November School and Workshop Excursion

Monday, 4 November

Anton Alekseev (Université de Genève)
From Poisson-Lie groups to inequalities and planar networks (3 lectures)

14:00 – 14:50 Lecture 1: *Eigenvalues, inequalities and planar networks*

Coffee / Tea Break

15:10 – 16:00 Lecture 2: *Poisson-Lie groups and linearization*

Coffee / Tea Break

16:30 – 17:20 Lecture 3: *Poisson-Lie groups and tropicalization*

School and Workshop Banquet

Tuesday, 5 November

Markus Pflaum (University of Colorado, Boulder)
Singular spaces and index theory (3 lectures)

14:00 – 14:50 Lecture 1: *Stratification theory of singular spaces*

Coffee / Tea Break

15:10 – 16:00 Lecture 2: *Groupoids*

Coffee / Tea Break

16:30 – 17:20 Lecture 3: *Index theory on groupoids and manifolds with boundaries*

Organizers: Jiang-Hua Lu & Siye Wu

Abstracts

Anton Alekseev (Université de Genève)
From Poisson-Lie groups to inequalities and planar networks (3 lectures)

1. *Eigenvalues, inequalities and planar networks*
2. *Poisson-Lie groups and linearization*
3. *Poisson-Lie groups and tropicalization*

Abstract: In the first lecture, we consider Gelfand-Zeiltin and Horn inequalities which come up in two completely different contexts: the first one is the theory of eigenvalues of Hermitian matrices, the second one is combinatorics of planar networks with Boltzmann weights on its edges. In the second lecture, we review the theory of Poisson brackets on Lie groups. One of the most important examples was pioneered by Lu and Weinstein. By the Ginzburg-Weinstein isomorphism theorem, this example admits a global linearization. In the last lecture, we explain how the formalism of Poisson-Lie groups developed in the second lecture helps to understand the surprizing link between Hermitian matrices and planar networks described in first lecture.

José M. Mourão (Instituto Superior Técnico, Lisboa)
Decomplexification of integrable systems, quantization and Kähler geometry (3 lectures)

1. *Quantization in Kähler and real polarizations*
2. *Imaginary time flow of Kähler structures and decomplexification*
3. *Imaginary time flow of holomorphic sections and tropicalization of divisors*

Abstract: We will give first some background on Kähler geometry, toric geometry and geometric quantization. Quantization on Kähler, real and mixed polarizations will be described together with illustrative examples. The problem of dependence of quantization on the choice of polarization will be discussed. We will study the geodesics on the infinite dimensional space of Kähler metrics on a (complex) projective manifold and their realization as imaginary time Hamiltonian flows – a formalism introduced in different contexts and with different goals by Semmes and Donaldson (Kähler geometry) and Thiemann (quantum theory). By considering the action on polarizations, rather than on Kähler forms, we will obtain the extension of geodesics to interesting non-Kähler polarizations. In some classes of Kähler manifolds we will describe geodesic rays degenerating to real polarizations and study the associated metric collapse. Each such ray selects a basis of holomorphic sections which converge to distributional sections supported on Bohr-Sommerfeld fibers as the geodesic time goes to infinity. We will review the tropicalization of amoebae of hypersurfaces on $(\mathbb{C}^*)^n$ and will show that the same geodesic rays give a natural extension of tropicalization to hypersurfaces on toric varieties.

Markus Pflaum (University of Colorado, Boulder)
Singular spaces and index theory (3 lectures)

1. *Stratification theory of singular spaces*
2. *Groupoids*
3. *Index theory on groupoids and manifolds with boundaries*

Abstract: The famous Atiyah-Singer index formula for closed manifolds has enriched the interaction between Analysis and Geometry/Topology and led to many new results in either of these areas. A topic of current mathematical research is the extension of the index formula to the singular setting. The series of three lectures will provide an introduction to this field. The concepts of stratified spaces and groupoids will be explained to build up the proper language to study the kind of singular spaces one mostly has in mind for applications in index theory. Then various analytic and geometric approaches to index theory in the singular setting will be explained. In particular we will concentrate on the noncommutative point of view, and show how it leads to the desired generalizations of the original index formula.