



Working Seminar on Several Complex Variables and Complex Geometry

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Rm 210, Run Run Shaw Building, HKU

From bounded symmetric domains to Fano manifolds: An introduction from the perspective of geometric structures

Abstract

Bounded symmetric domains are realizations of Hermitian symmetric manifolds of the semisimple and noncompact type. As Kähler manifolds they are of nonpositive sectional curvature and negative Ricci curvature, and they carry special geometric structures, e.g., projective structures, Grassmann structures and holomorphic conformal structures. Their compact duals are Hermitian symmetric manifolds of the semisimple and compact type such as projective spaces, Grassmannians and hyperquadrics.

We examine bounded symmetric domains Ω , their quotient manifolds $X_\Gamma = \Omega/\Gamma$, and the compact dual manifolds S of Ω from the analytic and geometric perspectives. A key element in their study is the presence of special geometric structures. In the case of the compact dual manifolds S , the underlying geometric structures called S -structures serve as prototypes for the geometric theory of uniruled projective manifolds (including especially Fano manifolds) based on VMRTs (varieties of minimal rational tangents) introduced by Hwang-Mok in the late 1990s. In the case of bounded symmetric domains, they relate to geometric questions on holomorphic isometries and analytic questions on proper holomorphic maps between these domains. For irreducible quotient manifolds $X_\Gamma = \Omega/\Gamma$ of finite volume of bounded symmetric domains Ω of rank ≥ 2 , the geometric structures are sources for metric rigidity. Moreover, holomorphic isometric embeddings of complex hyperbolic space forms are at the source of the study of Zariski closures of images of algebraic varieties under the universal covering map.