# Workshop on Complex Geometry

July 20 - 22, 2015

Room 210, Run Run Shaw Bldg., HKU

**Program and Abstracts**

*Institute of Mathematical Research*
*Department of Mathematics*

*Speakers:*

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|---|---|
| Meng Chen | Fudan U., Shanghai |
| Yik-Man Chiang | HKUST, Hong Kong |
| James Fullwood | HKU, Hong Kong |
| Yun Gao | SJTU, Shanghai & HKU, Hong Kong |
| Jaehyun Hong | Seoul National U., Seoul, Korea |
| Jun-Muk Hwang | KIAS, Seoul, Korea |
| Qingchun Ji | Fudan U., Shanghai |
| Zhi Jiang | Université de Paris-Sud, Orsay, France |
| Robert Lazarsfeld | Stony Brook U., USA |
| Sui-Chung Ng | East China Normal U., Shanghai |
| Yum-Tong Siu | Harvard U., USA |
| Shengli Tan | East China Normal U., Shanghai |
| Wing-Keung To | National U. Singapore, Singapore |
| Fei Ye | HKU, Hong Kong |
| Sai-Kee Yeung | Purdue U., USA |

***Organizers: Ngaiming Mok & Fei Ye***

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# Workshop on Complex Geometry

## July 20 - 22, 2015

## Room 210, Run Run Shaw Bldg., HKU

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| Time / Date | July 20 (Mon) | July 21 (Tue) | July 22 (Wed) |
|---|---|---|---|
| 9:30 – 10:30 | Siu | Lazarsfeld | Hwang |
| 10:30 – 10:50 | *Tea Break* | | |
| 10:50 – 11:50 | Ji | Fullwood | Hong |
| *Lunch Break* | | | |
| 14:00 – 15:00 | Chiang | Yeung | Jiang |
| 15:10 – 16:10 | Ng | Chen | Ye |
| 16:10 – 16:30 | *Tea Break* | | |
| 16:30 – 17:30 | Gao | To | Tan |

# Program

## July 20, 2015
## Monday

9:30 – 10:30 **Yum-Tong Siu**, Harvard U., USA
*Splitting of unstable plane bundle over the complex projective space*

*Tea Break*

10:50 – 11:50 **Qingchun Ji**, Fudan U., Shanghai
*Solvability of the Dirac equation and geometric applications*

*Lunch Break*

14:00 – 15:00 **Yik-Man Chiang**, HKUST, Hong Kong
*Malmquist-type theorems for the Painleve equations*

15:10 – 16:10 **Sui-Chung Ng**, East China Normal U., Shanghai
*On polarization technique and Segre varieties*

*Tea Break*

16:30 – 17:30 **Yun Gao**, SJTU, Shanghai & HKU, Hong Kong
*Proper holomorphic mappings among generalized balls*

# July 21, 2015
# Tuesday

9:30 – 10:30 **Robert Lazarsfeld**, Stony Brook U., USA
*Syzygies, gonality and symmetric products of curves*

*Tea Break*

10:50 – 11:50 **James Fullwood**, HKU, Hong Kong
*On tadpole relations via Verdier specialization*

*Lunch Break*

14:00 – 15:00 **Sai-Kee Yeung**, Purdue U., USA
*On surfaces of general type with maximal canonical degree*

15:10 – 16:10 **Meng Chen**, Fudan U., Shanghai
*On canonical stability index of projective varieties with large canonical volume*

*Tea Break*

16:30 – 17:30 **Wing-Keung To**, National U. Singapore, Singapore
*Effective aspects of certain stability property of positive Hermitian algebraic functions*

## July 22, 2015
## Wednesday

9:30 – 10:30 **Jun-Muk Hwang**, KIAS, Seoul, Korea
*Webs of algebraic curves*

*Tea Break*

10:50 – 11:50 **Jaehyun Hong**, Seoul National U., Seoul, Korea
*Fano varieties of cones over rational homogeneous varieties*

*Lunch Break*

14:00 – 15:00 **Zhi Jiang**, Université de Paris-Sud, Orsay, France
*Rational cohomology tori*

15:10 – 16:10 **Fei Ye**, HKU, Hong Kong
*Fujita's freeness conjecture on projective 5-folds*

*Tea Break*

16:30 – 17:30 **Shengli Tan**, East China Normal U., Shanghai
*Chern numbers of holomorphic foliations over an algebraic surface*

# Abstracts

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**Meng Chen**, Fudan U., Shanghai

*On canonical stability index of projective varieties with large canonical volume*

We study the canonical stability index of nonsingular projective varieties with large canonical volume. The key part is a special form of extension theorem which also applies to study varieties of large geometric genus. Some concrete results in dimensions 4 and 5 are obtained as direct applications. In the last part we propose some open questions and conjectures. This is a joint work with Zhi Jiang from Université Paris-Sud.

**Yik-Man Chiang**, HKUST, Hong Kong

*Malmquist-type theorems for the Painleve equations*

Malmquist was able to classify first-order algebraic equations which admit at least one meromorphic solution. Variant of this idea has been applied to solve differential equations with meromorphic solutions in a number of recent works of Eremenko, Conte, Ng, etc. In this joint work with Halburd, we show that there are second order analogues for the Malmquist problem. This extends earlier work of Wittich and Ishisaki.

**James Fullwood**, HKU, Hong Kong

*On tadpole relations via Verdier specialization*

S-duality between two regimes of string theory referred to as 'F-theory' and 'type IIB' predicts a linear relation among the Euler characteristic of an elliptic Calabi-Yau fourfold and the Euler characteristics of certain divisors in a particular Calabi-Yau threefold. Such relations are often referred to in the physics literature as 'tadpole relations'. It has been found that these tadpole relations coming from the equivalence between F-theory and type IIB may be shown to hold by integrating Chern class identities which hold in a much broader context than physical one. In this talk, using the construct of Verdier specialization we give a top-down explanation for the existence of such Chern class identities, yielding a purely mathematical explanation of the aforementioned tadpole relations predicted by physicists.

**Yun Gao**, SJTU, Shanghai & HKU, Hong Kong

*Proper holomorphic mappings among generalized balls*

We will introduce the fundamentals and literature about the proper holomorphic mappings among generalized balls. Generalized balls are symmetric domains on complex projective spaces whose boundaries are real hyperquadrics. The signature of the boundary real hyperquadric is the key issue in the study. We will explain the methods employed and talk about some classification results for certain special cases.

**Jaehyun Hong**, Seoul National U., Seoul, Korea

*Fano varieties of cones over rational homogeneous varieties*

Let $G$ be a connected reductive algebraic group over $\mathbb{C}$ and let $H$ be a closed subgroup. A homogeneous space $G/H$ is said to be *horospherical* if $H$ contains the unipotent radical of a Borel subgroup of $G$, or equivalently, $G/H$ is isomorphic to a torus bundle over a rational homogeneous variety $G/P$. A normal $G$-variety is called *horospherical* if it contains an open dense $G$-orbit isomorphic to a horospherical homogeneous space $G/H$. For example, toric varieties and rational homogeneous varieties are horospherical. Cones over rational homogeneous varieties are horospherical, too.

In this talk we study Fano varieties of cones over rational homogeneous varieties. Then we use them to give embeddings of smooth horospherical varieties of Picard number one as linear sections of rational homogeneous varieties.

**Jun-Muk Hwang**, KIAS, Seoul, Korea

*Webs of algebraic curves*

A family of algebraic curves covering a projective variety $X$ is called a web of curves on $X$ if it has only finitely many members through a general point of $X$. A web of curves on $X$ induces a web-structure, in the sense of local differential geometry, in a neighborhood of a general point of $X$. We will discuss the relation between the local differential geometry of the web-structure and the global algebraic geometry of $X$.

**Qingchun Ji**, Fudan U., Shanghai

*Solvability of the Dirac equation and geometric applications*

We study the (half) Dirac equation by Hörmander's $L^2$-method. By choosing appropriate weights, we get some geometric applications including eigenvalue estimates of the Dirac operator and "automatic transversality" of holomorphic curves with higher rank normal bundle and higher genus.

**Zhi Jiang**, Université de Paris-Sud, Orsay, France

*Rational cohomology tori*

We study normal compact Kähler spaces whose rational cohomology rings are isomorphic to that of a complex torus. We call them rational cohomology tori. First, we classify those with rational singularities up to dimension three. Then, we give constraints on the degree of the Albanese morphism and the number of simple factors of the Albanese variety for rational cohomology tori of general type (hence projective) with rational singularities. This is a joint work with Olivier Debarre and Marti Lahoz.

**Robert Lazarsfeld**, Stony Brook U., USA

*Syzygies, gonality and symmetric products of curves*

In the mid 1980s, Mark Green and I conjectured that one could read off the gonality of an algebraic curve $C$ from the syzygies among the equations defining any one sufficiently positive embedding of $C$. Ein and I recently noticed that a small variant of the ideas used by Voisin in her work on canonical curves leads to a quick proof of this gonality conjecture. The proof involves the geometry of certain vector bundles on the symmetric product of $C$. I will describe this circle of ideas, and I will also discuss a partial generalization, with Ein and Yang, to smooth varieties of all dimensions.

**Sui-Chung Ng**, East China Normal U., Shanghai

*On polarization technique and Segre varieties*

Polarization is a simple but very useful technique in Several Complex Variables and Complex Geometry. Namely, starting from an identity involving certain complex variables and their conjugates, one can obtain more identities by varying the conjugate variables independently. The resulting identities are then holomorphic in the original complex variables and are usually more powerful. The notion of Segre varieties came from polarization and they are the "polarized" real analytic varieties. In this talk, we will discuss how polarization and Segre varieties are useful in the rigidity of holomorphic mappings and Cauchy-Riemann mappings pertaining to various complex domains and CR manifolds.

**Yum-Tong Siu**, Harvard U., USA

*Splitting of unstable plane bundle over the complex projective space*

Will discuss a proof of the conjecture of Grauert-Schneider on the splitting of unstable holomorphic vector bundles of rank 2 over the complex projective space of dimension 4. The new techniques used in the proof requires the dimension of the complex projective space to be more than the originally conjectured dimension 4. The proof in the article posted on arXiv.org leaves out some situations which will be discussed in this talk.

**Shengli Tan**, East China Normal U., Shanghai

*Chern numbers of holomorphic foliations over an algebraic surface*

A holomorphic foliation $\mathcal{F}$ on a compact complex surface $X$ can be viewed as a global differential equation of the first order. The foliation or the differential equation is said to be algebraically integrable if all of the leaves are algebraic curves, equivalently, the differential equation has a rational first integral. In order to study differential equations of the first order, Poincaré suggested to study first the properties of the family of algebraic curves defined by a differential equation. The next step is to see which properties hold for the differential equations algebraically non-integrable. For example, the genus $g$ of the generic curves in the family is a topological invariants, Painlevé proposed the following problem: can we recognize the genus $g$ from the differential equation? Unfortunately, Lins Neto constructed some counterexamples to Painlevé's Problem. So the genus is not an invariant of the differential equations.

Our purpose of this talk is to prove that the modular Chern numbers of the family of curves are invariants of the differential equations. Therefore, we can introduce the Chern numbers $c_1^2(\mathcal{F})$, $c_2(\mathcal{F})$

and $\chi(\mathcal{F})$ for a holomorphic foliation $\mathcal{F}$, which are nonnegative rational numbers satisfying Noether's equality $c_1^2(\mathcal{F}) + c_2(\mathcal{F}) = 12\chi(\mathcal{F})$. These Chern numbers are birational invariants, and $c_1^2(\mathcal{F}) = 0$ iff $\mathcal{F}$ is not of general type. If the foliation $\mathcal{F}$ is algebraically integrable, then these invariants are exactly the modular Chern numbers of the family of curves defined by the rational first integral.

The birational classification of holomorphic foliations is almost completed by using the Kodaira dimensions. The Chern numbers can be used to get the biregular classification. As an application, we will give positive answers to the problems of Poincaré and Painlevé on the algebraic integrability of some foliations with small slopes $s(\mathcal{F}) = c_1^2(\mathcal{F})/\chi(\mathcal{F})$. We will also discuss the behavior of the pluricanonical systems of foliations of general type.

**Wing-Keung To**, National U. Singapore, Singapore

*Effective aspects of certain stability property of positive Hermitian algebraic functions*

In this talk, I will discuss some geometric approach to study a certain stability property of certain positive Hermitian algebraic functions on a compact complex manifold, which can be made effective in terms of certain geometric data of the Hermitian algebraic function. This is a joint work with Colin Tan.

**Fei Ye**, HKU, Hong Kong

*Fujitas freeness conjecture on projective 5-folds*

Let $X$ be a smooth projective variety of dimension 5 over $\mathbb{C}$ and $L$ be an ample line bundle on $X$. In this talk, I will introduce some new invariants and techniques, and explain difficulties and ideas in our proof of Fujita's freeness conjecture on $X$, i.e. the adjoint linear system $|K_X + kL|$ is base point free whenever $k \geq 6$.

**Sai-Kee Yeung**, Purdue U., USA

*On surfaces of general type with maximal canonical degree*

It has been conjectured that the maximal degree of the canonical map of a surface of general type is 36. The purpose of the talk is to explain results of myself and also a joint-work with Ching-Jui on this problem.