Workshop on Blaschke Products and Function Theory

July 29 - 31, 2015

Room 210, Run Run Shaw Bldg., HKU

Program and Abstracts

Institute of Mathematical Research
Department of Mathematics

Speakers:

Laurent Baratchart	INRIA-Sophia, France
Yang Chen	University of Macau, Macau
Anatoly Golberg	Holon Institute of Technology, Israel
Sergei Kalmykov	Far Eastern Federal University, Russia
Anna, Kit Ian Kou	University of Macau, Macau
Elijah Liflyand	Bar-Ilan University, Israel
Jinsong Liu	Chinese Academy of Sciences, Beijing
Javad Mashreghi	Université Laval, Canada
Alexander Müller-Hermes	Technical University Munich, Germany
Tao Qian	University of Macau, Macau
Oleg Szehr	Cambridge University, UK
Mikhail Tyaglov	Shanghai Jiao Tong University, Shanghai
Elias Wegert	TU Bergakademie Freiberg, Germany
Hasi Wulan	Shantou University, Shantou

Organizer: Tuen Wai Ng

Sponsor: Institute of Mathematical Research

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Time / Date	July 29 (Wed)	July 30 (Thur)	July 31 (Fri)
9:30 – 10:30		Mashreghi	Baratchart
10:30 – 10:50		*Tea Break*	
10:50 – 11:50	Qian	Chen	Wulan
Lunch Break			
14:00 – 15:00	Tyaglov	Szehr	Golberg
15:10 – 16:10	Kalmykov	Müller-Hermes	Liu
16:10 – 16:30	*Tea Break*		
16:30 – 17:30	Liflyand	Wegert	Kou

Program

July 29, 2015
Wednesday

10:00 – 10:45	***Reception at Rm 320A Run Run Shaw Bldg.***
10:50 – 11:50	**Tao Qian**, University of Macau, Macau *Boundary phase behavior of Blaschke products*

Lunch Break

14:00 – 15:00	**Mikhail Tyaglov**, Shanghai Jiao Tong University, Shanghai *Zeroes of finite differences of polynomials and entire functions*
15:10 – 16:10	**Sergei Kalmykov**, Far Eastern Federal University, Russia *Asymptotically sharp Bernstein type inequalities for polynomials on analytic arcs*

Tea Break

16:30 – 17:30	**Elijah Liflyand**, Bar-Ilan University, Israel *Fourier transform versus Hilbert transform*

July 30, 2015
Thursday

9:30 – 10:30 **Javad Mashreghi**, Université Laval, Canada
On mapping theorems for numerical range

Tea Break

10:50 – 11:50 **Yang Chen**, University of Macau, Macau
Topics in random matrices

Lunch Break

14:00 – 15:00 **Oleg Szehr**, Cambridge University, UK
Model theoretic methods and Nevanlinna-Pick interpolation in matrix analysis

15:10 – 16:10 **Alexander Müller-Hermes**, Technical University Munich, Germany
Spectral variation bounds via finite Blaschke products

Tea Break

16:30 – 17:30 **Elias Wegert**, TU Bergakademie Freiberg, Germany
Boundary Interpolation with Blaschke Products

July 31, 2015
Friday

9:30 – 10:30	**Laurent Baratchart**, INRIA-Sophia, France *On the rational approximation of Blaschke products*

Tea Break

10:50 – 11:50	**Hasi Wulan**, Shantou University, Shantou *QK and Morrey type spaces*

Lunch Break

14:00 – 15:00	**Anatoly Golberg**, Holon Institute of Technology, Israel *Local conformality conditions in plane and higher dimensions*
15:10 – 16:10	**Jinsong Liu**, Chinese Academy of Sciences, Beijing *Deformation of circle patterns and its applications*

Tea Break

16:30 – 17:30	**Anna, Kit Ian Kou**, University of Macau, Macau *Outer preserving in Hardy spaces*

Abstracts

Laurent Baratchart, INRIA-Sophia, France

On the rational approximation of Blaschke products

It is easy to see that infinite Blaschke products are badly approximable by rational functions in H^∞. That is, the best approximation by a rational function of degree n to a given infinite Blaschke product in uniform norm on the circle is zero. In contrast, they are approximable in L^p norms, $p < \infty$, in the sense that the approximation error will tend to zero as n goes large. In this talk, we give a lower estimate on the speed of convergence in terms of the zeros of the Blaschke product. The estimate rests on a min-max principle of Courant type for singular numbers of Hankel operators, which also involves the geometry of Blaschke product in its proof.

Yang Chen, University of Macau, Macau

Topics in random matrices

In this talk I will present a range of problems arising from Random Matrix Theory; singular deformation of Jacobi weight, generating functions of linear statistics with Normal and Gamma background, and small eigenvalues of large Hankel matrices.

Anatoly Golberg, Holon Institute of Technology, Israel

Local conformality conditions in plane and higher dimensions

There are various necessary conditions for local conformality (that is comformality of mappings at a point) in the complex plane, like preservation of angles, independence of stretching on direction, asymptotic homogeneity, circle-like behavior, quasisymmetry, etc. Each of those can be treated as a local weak conformality condition. In higher dimensions, the situation becomes rather rigid, since due to the classical Liouville theorem, conformal mappings even in $\mathbb{R}^3$ are reduced to the Möbius transformations, i.e. to finite compositions of reflections across the spheres. On the other hand, many classes of mappings, e.g. quasiconformal mappings, mappings of finite distortion, mappings with controlled moduli admit differentiability almost everywhere and Hölder continuity, which also can be regarded as the weak conformality conditions.
In the talk, we discuss various aspects of local weak conformality, their relations and provide the corresponding sufficient conditions. All these results can be regarded as extensions of the classical Teichmüller-Wittich-Belinskiĭ theorem.

Sergei Kalmykov, Far Eastern Federal University, Russia

Asymptotically sharp Bernstein type inequalities for polynomials on analytic arcs

Bernstein (or Riesz) type polynomial inequalities are well known. On the complex plane Bernstein inequality was extended to compact sets bounded by smooth Jordan curves and the asymptotically sharp constant can be expressed via the normal derivative of Green's function [1]. There is a general conjecture by Totik for Jordan arcs that the asymptotically sharp Bernstein factor can be expressed

as the maximum of the two normal derivatives of Green's function. It was proved for the subarcs [2], and later, for general subsets of the unit circle [3]. In this talk we consider the case of an arbitrary analytic Jordan arc. The proofs of the main results (see [4]) are based on facts from potential and interpolation theories, Borwein-Erdélyi inequality for derivative of rational functions on the unit circle, Gonchar-Grigorjan estimate of the norm of the holomorphic component of meromorphic function, Totik's construction of fast decreasing polynomials, and conformal mappings.
This is based on a joint work with Béla Nagy.

[1] B. Nagy, V. Totik, Sharpening of Hilbert's lemniscate theorem, Journal d'Analyse Mathematique, 2005, 96, 191-223.

[2] B. Nagy, V. Totik, Bernstein's inequality for algebraic polynomials on circular arcs, Constructive Approximation, 2013, 37(2), 2013, 223–232.

[3] B. Nagy, V. Totik, Riesz-type inequalities on general sets, JMAA, 2014, 416(1), 344-351.

[4] S.I. Kalmykov, B. Nagy, Polynomial and rational inequalities on Jordan arcs and domains, JMAA, 2015, 430(2), 874-894.

Anna, Kit Ian Kou, University of Macau, Macau

Outer preserving in Hardy spaces

The goal of this talk is to characterize the semigroup of bounded linear operators on the Hardy space $H^p(D)$ that preserve the set of shifted outer functions. The outer functions play a crucial role in the Beurling factorization, where every function in the Hardy space can be expressed as a product of outer and inner functions, the inner functions being the part containing the zeros. A complete description of this semigroup of operators is given. This work is motivated by digital signal processing in the context of geophysical imaging. The class of shifted outer functions represents delayed, causal, minimum-phase signals that model impulsive physical sources.

Elijah Liflyand, Bar-Ilan University, Israel

Fourier transform versus Hilbert transform

We present several results in which the interplay between the Fourier transform and the Hilbert transform is of special form and importance.

1. In 50-s (Kahane, Izumi-Tsuchikura, Boas, etc.),the following problem in Fourier Analysis attracted much attention:
Let $\{a_k\}$, $k = 0, 1, 2...$, be the sequence of the Fourier coefficients of the absolutely convergent sine (cosine) Fourier series of a function $f : \mathbb{T} = [-\pi, \pi) \to \mathbb{C}$, that is $\sum |a_k| < \infty$. Under which conditions on $\{a_k\}$ the re-expansion of $f(t)$ ($f(t) - f(0)$, respectively) in the cosine (sine) Fourier series will also be absolutely convergent?
We solve a similar problem for functions on the whole axis and their Fourier transforms. Generally, the re-expansion of a function with integrable cosine (sine) Fourier transform in the sine (cosine) Fourier transform is integrable if and only if not only the initial Fourier transform is integrable but also the Hilbert transform of the initial Fourier transform is integrable.

2. The following result is due to Hardy and Littlewood:
If a (periodic) function f and its conjugate $\widetilde{f}$ are both of bounded variation, their Fourier series converge absolutely. We generalize the Hardy-Littlewood theorem (joint work with U. Stadtmüller) to the Fourier transform of a function on the real axis and its modified Hilbert transform. The initial

Hardy-Littlewood theorem is a partial case of this extension, when the function is taken to be with compact support.

3. These and other problems are integrated parts of harmonic analysis of functions of bounded variation. We have found the maximal space for the integrability of the Fourier transform of a function of bounded variation.Along with those known earlier, various interesting new spaces appear in this study. Their inter-relations lead, in particular, to improvements of Hardy's inequality. There are multidimensional generalizations of these results.

Jinsong Liu, Chinese Academy of Sciences, Beijing

Deformation of circle patterns and its applications

Given a circle pattern on the Riemann sphere $\hat{\mathbb{C}}$, in this talk we prove that its quasiconformal deformation space can be naturally identified with the product of the Teichmüller spaces of its interstices. By using the intersection number technique, together with Teichmüller theory of packings, we provides an alternative approach to the Midscribability Theorem. Furthermore, by combining Schramm's method with the above ones, we obtain a rigidity result as well. Furthermore, by using these methods, we shall investigate the stability of some inscribable graphs.

Javad Mashreghi, Université Laval, Canada

On mapping theorems for numerical range

Let T be an operator on a Hilbert space H with numerical radius $w(T) \le 1$. According to a theorem of Berger and Stampfli, if f is a function in the disk algebra such that $f(0) = 0$, then $w(f(T)) \le \|f\|_\infty$. We give a new and elementary proof of this result using finite Blaschke products.
A well-known result relating numerical radius and norm says $\|T\| \le 2w(T)$. We obtain a local improvement of this estimate, namely, if $w(T) \le 1$ then

$$\|Tx\|^2 \le 2 + 2\sqrt{1 - |\langle Tx, x\rangle|^2} \qquad (x \in H,\ \|x\| \le 1).$$

Using this refinement, we give a simplified proof of Drury's teardrop theorem, which extends the Berger–Stampfli theorem to the case $f(0) \ne 0$.

Joint work with T. Ransford and H. Klaja.

Alexander Müller-Hermes, Technical University Munich, Germany

Spectral variation bounds via finite Blaschke products

We derive new estimates for distances between optimal matchings of eigenvalues of non-normal matrices in terms of the norm of their difference. We introduce and estimate a hyperbolic metric analogue of the classical spectral variation distance. Our apporach is based on the theory of model operators, which provides strong resolvent estimates. The latter naturally lead to a Chebychev-type interpolation problem with finite Blaschke products, which can be solved explicitly. Our bound improves on the best known classical spectral variation bounds if the distance of matrices is sufficiently small and it is sharp for asymptotically large matrices.

Tao Qian, University of Macau, Macau

Boundary phase behavior of Blaschke products

It is well known that the phase derivative of a Möbius transform on the boundary is, by module a multiple constant, the Poisson kernel. Therefore, it, in particular, has the positivity property. While finite Blaschke products are trivial, the talk will show that this positive phase derivative property can be extended to infinite Blaschke products through the Wolff-Julia-Caratheodory theorem in 1930's with the formulation of non tangential boundary limit. The talk then includes a quick survey on impacts of this result to signal analysis, especially to DSP (digital signal processing). The talk finishes with providing information of latest studies on finding analytic functions with positive boundary phase derivatives.

Oleg Szehr, Cambridge University, UK

Model theoretic methods and Nevanlinna-Pick interpolation in matrix analysis

One of the most basic tasks in matrix analysis is to find a spectral estimate to the norm of a function of a matrix. Examples of this task abound in ubiquitous forms, e.g. as bounds on condition numbers in (numerical) stability analysis, as convergence estimates in the theory of Markov chains and as so-called spectral-variation estimates in theoretical matrix analysis. In this talk we present a method that relates the problem of finding an eigenvalue bound to a function of an operator of certain class to a Nevanlinna-Pick interpolation problem in an associated function algebra. This method draws on deep results from Fourier analysis, interpolation theory and the theory of Hilbert function spaces (e.g. Sarasons's commutant lifting theorem) and allows us to improve on known spectral estimates in the mentioned examples. For the class of operators on Hilbert space whose norm is bounded by 1 (i.e. contractions) this method provides us with a complete solution to the problem of finding a spectral estimate. A crucial role in this solution is played by the so-called model operator, which is the compression of a backward shift operator on the Hardy space H^2 to an invariant subspace. We contribute to the theory of such operators by providing explicit matrix representations.

Mikhail Tyaglov, Shanghai Jiao Tong University, Shanghai

Zeroes of finite differences of polynomials and entire functions

Some classes of finite differences that preserve roots of univariate polynomials on lines or in strips and half-planes of the complex plane will be presented. In particular, we describe some classes of finite differences that preserve the hyperbolicity (real-rootedness) of polynomials and prove a finite difference analogue of the Hermite-Pauline theorem (completely different from the one recently established by Brändén, Krasikov and Shapiro). We also found the polynomial whose finite differences has the minimal mesh (minimal distance between roots) among all other polynomials. Corresponding results for entire functions will be presented. Finally, some asymptotic results for roots of finite differences of polynomials will be presented.

Elias Wegert, TU Bergakademie Freiberg, Germany

Boundary Interpolation with Blaschke Products

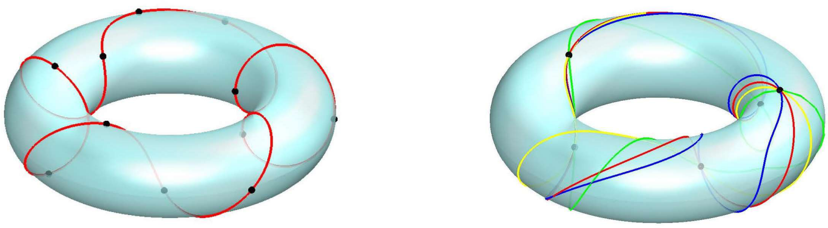
The classical interpolation problem of Nevanlinna and Pick consists in finding unimodularly bounded holomorphic functions in the complex unit disk $\mathbb{D}$ which satisfy the interpolation conditions

$$f(z_k) = w_k, \qquad k = 1, \ldots, n, \tag{1}$$

at given points $z_1, \ldots, z_n \in \mathbb{D}$. Criteria for the solvability of (1) are well known for about one hundred years.

What happens if the nodes z_k and the values w_k are located on the unit circle? Surprizingly late (in 1987) Ruscheweyh and Jones showed that then (1) is always solveable by a Blaschke product of degree less than or equal to $n-1$. For some problems the minimal degree is in fact $n-1$ – but not for all.

In the lecture we show that all boundary interpolation problems of the considered type fall into three classes which are distinguished by the minimal degree of the interpolating function. Only one of these classes consists of well-posed problems.



Visualization of two problems with minimal solutions

We propose an algorithm for classifying a given boundary interpolation problem, which also allows to find the minimal degree of the solution for generic problems, and discuss some open questions. This is a joint with Gunter Semmler.

Hasi Wulan, Shantou University, Shantou

QK and Morrey type spaces

In this talk, some recent results on QK spaces and Morrey type spaces, including the decomposition theorem, the fractional order derivatives characterization and the pseudoanalytic extensions, will be given. A relationship between QK spaces and Morrey type spaces will be mentioned.