THE UNIVERSITY OF HONG KONG

Institute of Mathematical Research
Department of Mathematics

COLLOQUIUM

Billiard on the triaxial ellipsoid

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Abstract

The triaxial ellipsoid seems on the first sight an extremely trivial object. Topologically it is just a sphere and there is not much to say about it. However at least since Jacobi we know that this is not the case. Jacobi has shown in his wonderful monograph *Vorlesungen über Dynamik*, Gesammelte Werke 8, how rich the mathematics around the ellipsoid is. One of the key object is the pencil of confocal ellipsoids

$$\frac{x^2}{a-\lambda}+\frac{y^2}{b-\lambda}+\frac{z^2}{c-\lambda}=1$$

with $0 < a < b < c$ and $\lambda \in \mathbb{R}$. Jacobi introduced the famous *elliptic coordinates* which are nowadays indispensable for geodesists and earth scientists. It is further of great importance for studying the differential geometry of the ellipsoid. As an outcome of these studies a large variety of elliptic and hyperelliptic curves became visible and with that the theory of dynamical and Hamiltonian systems entered. One of the foci were the so-called principal curvature lines which appear as the intersection of two such ellipsoids in the pencil. Much more complicated were the genus two hyperelliptic curves which are intimately related to the geodesics on the ellipsoid which turn out to have a very strange topological property. In particular they are lines on the ellipsoid which in general wind around without getting closed in finite time. In a joint project with Ronaldo Garcia we looked at the problem of characterizing closed geodesics in terms of the geometry of the associated abelian surface which is the Jacobian of some genus 2 hyperelliptic curve. To our knowledge so far no curvature line which takes a non-zero angle with respect to one of the principal curvature lines has been shown to be not closed. It is expected they are not closed in general so that a billiard ball turning around along these lines never comes back. Both of these problems lead to abelian integrals, in the curvature lines case to an elliptic integral of the third kind and in the geodesic situation to an abelian integral of the first kind and the questions can be answered by solving two of the problems of Th. Schneider about the transcendence of such integrals. In both cases one has to determine the structure of the endomorphism algebra of special commutative algebraic groups. This is not so difficult (surprise-surprise!) in the case of geodesics. In the case of the curvature lines the algebraic group is an extension of an elliptic curve by a product of additive and multiplicative groups and here things become much more complicated.

Date:	**April 26, 2016 (Tuesday)**
Time:	**4:00 – 5:00pm**
Place:	**Room 210, Run Run Shaw Bldg., HKU**

All are welcome