THE UNIVERSITY   **OF HONG KONG**

Institute of Mathematical Research
Department of Mathematics

Geometry Seminar

Negative Weight Derivation and Rational Homotopy Theory

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Abstract

In the spring of 1976, Halperin proposed his famous conjecture in the rational homotopy seminar in Lille. If F is an elliptic space with positive Euler characteristic and $F \to E \to B$ is a Serre fibration of simply connected spaces, then the Serre spectral sequence for this fibration collapses at E_2 term. This conjecture remains open. Meier proved that collapsing of the Serre spectral sequence is closely related to vanishing of derivations of the cohomology algebra of F. Here, we formulate two conjectures. The first conjecture generalizes Halperin Conjecture in that the fiber F is no longer required to be an elliptic space with positive Euler characteristic.

Conjecture I: Suppose the cohomology algebra of X is a finite dimensional commutative positively graded algebra of the form $R = C[x_1, ..., x_n]/(f_1, ..., f_m)$. If the weight of each x_i is a positive even integer and weight degree of each f_i is bounded below by a suitable constant C (The constant C depends only on weights of $x_1, ..., x_n$), then the Serre spectral sequence of any fibration with fiber X collapses.

Conjecture I is a consequence of the following Conjecture II.

Conjecture II: Let $R = C[x_1, ..., x_n]/(f_1, ..., f_m)$ be a finite dimensional commutative positively graded algebra. If weight degree of each f_i is bounded below by a suitable constant C (The constant C depends only on weights of $x_1, ..., x_n$), then all derivations of negative degree of R vanish.

In this lecture, we prove that Conjectures I and II are true. This is a joint work with Hao Chen and Huaiqing Zuo.

Date:	May 4, 2016 (Wednesday)
Time:	11:00am – 12:00noon
Place:	Room 210, Run Run Shaw Bldg., HKU

All are welcome