Unique ergodicity for foliations in $\mathbb{P}^2$ with an invariant curve

Consider the polynomial differential equation in $\mathbb{C}^2$

$$\frac{dz}{dt} = P(z, w), \quad \frac{dw}{dt} = Q(z, w).$$

The polynomials $P$ and $Q$ are holomorphic, the time is complex. In order to study the global behavior of the solutions, it is convenient to consider the extension as a foliation in the projective plane $\mathbb{P}^2$.

Assume that the foliation is generic in the sense that its singular points are hyperbolic and that the line at infinity is the unique invariant algebraic curve. In this context, Khudai-Veronov has shown that except for the line at infinity all leaves are dense. This follows from the study of the holonomy on the invariant line.

In a recent work with T.C. Dinh we show that there is a unique positive harmonic $(1, 1)$-current of mass 1 which is directed by the foliation and this is the current of integration on the invariant line. In a point of view from Nevanlinna’s theory, every leaf of the foliation is concentrated near the invariant line. Although leaves are dense. The result is based on a new geometric method, the density of a current along a curve.

Tea Break

Higgs-de Rham flow and applications

In this talk, I would like to explain the notion “Higgs-de Rham flow” in char $p$ and mixed char, introduced by Lan, Zuo and myself. Known applications include: i) construction of two dimensional crystalline representations of arithmetic fundamental groups of certain hyperbolic curves over the Witt ring, recovering the main result of S. Mochizuki on ordinary curves; ii) a char $p$ proof of Bogomolov-Gieseker’s inequality for semistable Higgs bundles and Miyaoka-Yau’s chern number inequalities for algebraic surfaces, obtained by Adrian Langer.

This meeting is hosted by the Institute of Mathematical Research, HKU.

All are Welcome