Abstract

As is well-known one of the first appearances of linear algebraic groups was as differential Galois groups of linear differential equations, or connections. In the case of regular singular connections the differential Galois group is closely related to the monodromy group, i.e. the fundamental group representation determined by the connection. One can ‘upgrade’ from the differential Galois group to the monodromy representation, and use this to classify connections, i.e. to describe the moduli space of connections as a variety. Such varieties (known as character varieties or Betti spaces) have an exceedingly rich geometry and dynamics. The aim of these lectures is to describe what happens when we consider irregular connections, and the resulting Stokes data enriching the fundamental group representation. All of the features of the tame case generalise nicely. Moreover several aspects of the theory of linear algebraic groups appear in this context, and so can be given a modular interpretation, as will be explained.