THE UNIVERSITY  OF HONG KONG

Institute of Mathematical Research
Department of Mathematics

WORKSHOP ON RANDOM MATRICES

Friday, June 16, 2017

Room 210, Run Run Shaw Building, HKU

9:30 – 9:35	Ngaiming Mok (Director of IMR, HKU) *Opening remarks*
9:35 – 10:35	Opening Lecture Jon Keating, University of Bristol, UK *Extreme value statistics: from random matrices to number theory*
Coffee / tea break	
10:55 – 11:40	Xiang-Dong Li, Chinese Academy of Sciences *On the law of large numbers and functional central limit theorem for generalized Dyson Brownian motion*
11:40 – 12:25	Zhigang Bao, HKUST *Local single ring theorem on optimal scale*
Lunch Break	
14:00 – 14:45	Dong Wang, National University of Singapore *One dimensional free fermions at finite temperature and the MNS matrix model*
14:45 – 15:30	Jianfeng Yao, HKU *On structure testing for component covariance matrices of a high-dimensional mixture*
Coffee / tea break	
15:45 – 16:30	Tuen-Wai Ng, HKU *Critical points of random polynomials*
16:30 – 17:15	Guangming Pan, Nanyang Technological University *Central limit theorems for the unbounded spiked eigenvalues and the largest non-spike of sample covariance matrices*

Organizers: Ngaiming Mok, Tuen-Wai Ng, Jianfeng Yao

Abstracts

Zhigang Bao, Department of Mathematics, Hong Kong University of Science and Technology

Local single ring theorem on optimal scale

The celebrated single ring theorem asserts that the eigenvalues of a non-Hermitian random matrix with given singular values are asymptotically distributed on a single ring in the complex plane. In this talk, I will present a local version of this theorem on the optimal scale, which implies the optimal convergence rate of the single ring law. This is a joint work with Laszlo Erdos and Kevin Schnelli.

Jon Keating, School of Mathematics, University of Bristol, UK

Extreme value statistics: from random matrices to number theory

I will review some of the (many) recent developments concerning the extreme value statistics of the characteristic polynomials of random matrices, and connections to the theory of the Riemann zeta function.

Xiang-Dong Li, Academy of Mathematics and Systems Science, Chinese Academy of Sciences

On the Law of Larger Numbers and Functional Central Limit Theorem for Generalized Dyson Brownian Motion

We study the generalized Dyson Brownian motion (GDBM) of an interacting N-particle system with logarithmic interaction and general external potential V. Under reasonable condition on V , we prove the existence and uniqueness of strong solution to SDE for GDBM. We then prove that the family of the empirical measures of GDBM is tight on $C([0,T],P(R))$ and all the large N limits satisfy a nonlinear McKean-Vlasov equation. Inspired by previous works due to Biane and Speicher, Carrillo, McCann and Villani, we prove that the McKean-Vlasov equation is indeed the gradient flow of the Voiculescu free entropy on the Wasserstein space of probability measures over R. Using the optimal transportation theory, we prove that if $V'' \geq K$ for some constant $K \in R$, the McKean-Vlasov equation has a unique weak solution. This proves the Law of Large Numbers and the propagation of chaos for the empirical measures of GDBM. Finally, we prove the Functional Central Limit Theorem for the empirical measure of GDBM towards the McKean-Vlasov equation. Joint work with Songzi Li and Yongxiao Xie.

Tuen-Wai Ng, Department of Mathematics, The University of Hong Kong

Critical points of random polynomials

The study of zero distribution of random polynomials has a long history and is currently a very active research area. Traditionally, the randomness in these polynomials comes from the probability distribution followed by their coefficients. One can introduce randomness in the zeros (instead of the coefficients) of polynomials, and then investigate the locations of their critical points (relative to these zeros). Such a study was initiated by Rivin and the late Schramm in 2001, but only until 2011, Pemantle and Rivin proposed a precise probabilistic framework of it which will first be explained in this talk. Following this framework, we will consider the problem of finding the zero distributions of

the derivatives of random polynomials with i.i.d. zeros following a common distribution supported on a subset of the complex plane. This is a joint work with Pak-Leong Cheung, Jonathan Tsai and Phillip Yam.

Guangming Pan, School of Physical & Mathematical Sciences, Nanyang Technological University

Central Limit Theorems for the Unbounded Spiked Eigenvalues and the largest non-spike of Sample Covariance Matrices

Consider a spiked population covariance matrix with the first K largest eigenvalues $\mu_1, \cdots, \mu_K$ tending to infinity (spikes) and the remaining eigenvalues being bounded. We establish the asymptotic joint distributions for the spiked eigenvalues of its sample covariance matrices when the population spikes satisfy $\mu_i = \frac{p}{nd_i}$ with $d_i \to 0, i = 1, 2, \cdots, K$and $\liminf_{n\to\infty} \frac{p}{n} > 0$. The number of the spikes K is allowed to diverge with a certain rate. Further more, the largest non spiked eigenvalue is also shown to converge in distribution to Type-1 Tracy-Widom distribution under some mild conditions. We also estimate the number of the spiked eigenvalues.

Dong Wang, Department of Mathematics, National University of Singapore

One dimensional free fermions at finite temperature and the MNS matrix model

In this talk we discuss the local statistics of free fermions on the real line, in a quadratic potential well, and at finite temperature. This free fermion model is equivalent to a matrix model defined by Moshe, Neuberger and Shapiro, and our result also gives the local statistics of eigenvalues of the MNS matrix model. The MNS matrix model has a non-universal feature that distinguishes it from other matrix models.
This is joint work with Karl Liechty.

Jianfeng Yao, Department of Statistics and Actuarial Science, The University of Hong Kong

On structure testing for component covariance matrices of a high-dimensional mixture

By studying the family of p-dimensional scale mixtures, this paper shows for the first time a non trivial example where the eigenvalue distribution of the corresponding sample covariance matrix *does not converge* to the celebrated Marčenko-Pastur law. A different and new limit is found and characterized. The reasons of failure of the Marčenko-Pastur limit in this situation are found to be a strong dependence between the p-coordinates of the mixture. Next, we address the problem of testing whether the mixture has a spherical covariance matrix. To analize the traditional John's type test we establish a novel and general CLT for linear statistics of eigenvalues of the sample covariance matrix. It is shown that the John's test and its recent high-dimensional extensions both fail for high-dimensional mixtures, precisely due to the different spectral limit above. As a remedy, a new test procedure is constructed afterwards for the sphericity hypothesis. This test is then applied to identify the covariance structure in model-based clustering. It is shown that the test has much higher power than the widely used ICL and BIC criteria in detecting non spherical component covariance matrices of a high-dimensional mixture.
This is a joint work with Li Weiming.