Workshop on Stochastic Analysis and Related Topics

July 3 - 5, 2018

Room 210, Run Run Shaw Bldg., HKU

Program and Abstracts

Institute of Mathematical Research
Department of Mathematics

Speakers:

Jiro Akahori	Ritsumeikan University
Guangyue Han	The University of Hong Kong
Yaozhong Hu	University of Alberta
Davar Khoshnevisan	University of Utah
Arturo Kohatsu-Higa	Ritsumeikan University
Jin Ma	University of Southern California
Kihun Nam	Monash University
Lluís Quer-Sardanyons	Universitat Autònoma Barcelona
Xiaoming Song	Drexel University
Samy Tindel	Purdue University
Ciprian Tudor	Université Paris 1
Tai-Ho Wang	Barach College, City University of New York
Jing Wu	Sun Yat-Sen University
Panqiu Xia	University of Kansas
George Yuan	Soochow University & BBD Inc.
Jianfeng Zhang	University of Southern California
Xicheng Zhang	Wuhan University

Organizing Committee:

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Stochastic Analysis and Related Topics

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Time / Date	July 3 (Tue)	July 4 (Wed)	July 5 (Thur)
9:30 – 10:30	Jin Ma	Arturo Kohatsu-Higa	Davar Khoshnevisan
10:30 – 10:50	*Tea Break*		
10:50 – 11:50	Xicheng Zhang	George Yuan	Samy Tindel
Lunch Break			
14:00 – 15:00	Yaozhong Hu	Jianfeng Zhang	Jiro Akahori
15:10 – 16:10	Tai-Ho Wang	Lluís Quer-Sardanyons	Jing Wu
16:10 – 16:30	*Tea Break*		
16:30 – 17:30	Kihun Nam	Ciprian Tudor	Xiaoming Song
17:30 – 18:30	Guangyue Han	Panqiu Xia	

Jul 3, 2018

Program

July 3, 2018
Tuesday

9:30 – 10:30 **Jin Ma**, University of Southern California
Time Consistent Conditional Expectation under Probability Distortion

Tea Break

10:50 – 11:50 **Xicheng Zhang**, Wuhan University
Dirichlet Problem for Supercritical Non-local Operators

Lunch Break

14:00 – 15:00 **Yaozhong Hu**, University of Alberta
Drift Parameter Estimator in Linear and Nonlinear Stochastic Differential Equation Driven by Fractional Brownian Motions

15:10 – 16:10 **Tai-Ho Wang**, Barach College, City University of New York
Target Volatility Option Pricing in Lognormal Fractional SABR Model

Tea Break

16:30 – 17:30 **Kihun Nam**, Monash University
Fixed Point Formulation for Backward SDEs and their Generalizations

17:30 – 18:30 **Guangyue Han**, The University of Hong Kong
Introduction to Information Theory

July 4, 2018
Wednesday

9:30 – 10:30 **Arturo Kohatsu-Higa**, Ritsumeikan University
IBP for Stopped Processes

Tea Break

10:50 – 11:50 **George Yuan**, Soochow University & BBD Inc.
The Dynamics of Stochastic Incentive Effect for "U" Shape Theory for SMEs under Bigdata Framework

Lunch Break

14:00 – 15:00 **Jianfeng Zhang**, University of Southern California
A Martingale Approach for Fractional Brownian Motions and Related Path Dependent PDEs

15:10 – 16:10 **Lluís Quer-Sardanyons**, Universitat Autònoma Barcelona
Existence of Density for the Stochastic Wave Equation with Space-time Homogeneous Noise

Tea Break

16:30 – 17:30 **Ciprian Tudor**, Université Paris 1
Correlation Structure and Quadratic Variations of the Solution to the Wave Equation

17:30 – 18:30 **Panqiu Xia**, University of Kansas
Joint Holder Continuity of the Solutions to a Class of SPDEs Arising from Multi-dimensional Superprocesses in the Random Environment

July 5, 2018
Thursday

9:30 – 10:30	**Davar Khoshnevisan**, University of Utah *Global Solutions to Reaction-Diffusion Equations with Superlinear Drift and Multiplicative Noise*

Tea Break

10:50 – 11:50	**Samy Tindel**, Purdue University *Discrete Rough Paths and Limit Theorems*

Lunch Break

14:00 – 15:00	**Jiro Akahori**, Ritsumeikan University *Stochastic Volatility Models of Quadratic Volterra Gaussian Type*
15:10 – 16:10	**Jing Wu**, Sun Yat-Sen University *Limit Theorems for Multivalued Stochastic Differential Equations*

Tea Break

16:30 – 17:30	**Xiaoming Song**, Drexel University *Probability Density of Lognormal Fractional SABR Model*

Jiro Akahori, Ritsumeikan University

Stochastic Volatility Models of Quadratic Volterra Gaussian Type

In the talk I will introduce first a class of asset price models where state variables are given by (infinite dimensional) quadratic Gaussian processes, and show that (some of) classical Heston models belong to the class. We then introduce a "fractional volatility" extension of Heston model within the class and discuss some mathematical properties. (joint work with Xiaoming Song, and Tai-Ho Wang)

Guangyue Han, The University of Hong Kong

Introduction to Information Theory

Information theory is a branch of applied mathematics and electrical engineering involving the quantification of information. Historically, information theory was developed by Claude E. Shannon to find fundamental limits on signal processing operations such as compressing data and on reliably storing and communicating data. Since its inception it has broadened to find applications in many other areas, including statistical inference, artificial intelligence, cryptography, biology, statistical physics, quantum computing, computational complexity, data analysis and so on. This talk is to give a brief introduction to this rapidly developing theory.

Yaozhong Hu, University of Alberta

Drift Parameter Estimator in Linear and Nonlinear Stochastic Differential Equation Driven by Fractional Brownian Motions

We derive the strong consistency of the least squares estimator for the drift coefficient of a fractional stochastic differential system. The drift coefficient is one-sided dissipative Lipschitz and the driving noise is additive that continuous observation is possible. The main tools are ergodic theorem and Malliavin calculus. As a by-product, we derive a maximum inequality for Skorohod integrals, which plays an important role to obtain the strong consistency of the least squares estimator. This is from some joint work with David Nualart and Hongjuan Zhou.

Davar Khoshnevisan, University of Utah

Global Solutions to Reaction-Diffusion Equations with Superlinear Drift and Multiplicative Noise

Let ξ denote space-time white noise and consider a reaction-diffusion equation of the form

$$\dot{u}(t\,,x) = \tfrac{1}{2}u(t\,,x) + b(u(t\,,x)) + \sigma(u(t\,,x))\xi(t\,,x),$$

on $\mathbb{R}_+ \times [0\,,1]$ with homogeneous Dirichlet boundary conditions and suitable initial data, in the case that there exists $\varepsilon > 0$ such that $|b(z)| \geq |z|(\log|z|)^{1+\varepsilon}$ for all sufficiently-large

values of $|z|$. When $\sigma \equiv 0$, it is well known that such PDEs frequently have non-trivial stationary solutions. By contrast, Bonder and Groisman (2009) have recently shown that there is finite-time blowup when σ is a non-zero constant. In this paper, we prove that the Bonder-Groisman condition is unimproveable by showing that the reaction-diffusion equation with noise is "typically well posed when $|b(z)| = O(|z| \log_+ |z|)$ as $|z| \to \infty$. We interpret the word "typically in two essentially-different ways without altering the conclusions of our assertions.
This is based on joint work with Robert C. Dalang and Tusheng Zhang.

Arturo Kohatsu-Higa, Ritsumeikan University

IBP for Stopped Processes

We present the integration by parts formula for stopped processes based on Markov chain approximations of the limit process.
This is joint work with Noufel Frikha (Paris-Diderot) and Libo Li (University New South Wales).

Jin Ma, University of Southern California

Time Consistent Conditional Expectation under Probability Distortion

We introduce a new notion of conditional nonlinear expectation in the case where the underlying probability scale is distorted by a weight function. Such a distorted nonlinear expectation is non-sub-additive in general, hence beyond the scope of Peng's well-known framework of nonlinear expectations. A more fundamental problem when extending such distorted expectation to a dynamic setting is the time inconsistency, that is, the usual "tower property" fails. We show that, by localizing the probability distortion and restricting to a smaller class of random variables, it is possible to construct a conditional expectation is such a way that it coincides with the original nonlinear expectation at time zero, but it also has a time-consistent dynamics in the sense that the tower property remains valid. Furthermore, we show that this conditional expectation can be associate to a partial differential equation (hence even a backward stochastic dierential equation), which involves the law of the underlying diusion. This work is the rst step towards a new understanding of nonlinear expectations beyond capacity theory, and will potentially be a helpful tool for solving time inconsistent stochastic optimization problems.
This is a joint work with Ting-Kam Leonard Wong and Jianfeng Zhang.

Kihun Nam, Monash University

Fixed Point Formulation for Backward SDEs and their Generalizations

In this talk, we introduce a class of backward stochastic equations (BSEs) that extend classical BSDEs and include many interesting examples of generalized BSDEs as well as semimartingale backward equations. We show that a BSE can be translated into a fixed point problem in a space of random vectors. This makes it possible to employ general fixed point arguments, either algebraic or topological to find solutions. For instance, Banach's contraction mapping theorem can be used to derive general existence and uniqueness results for equations with Lipschitz coefficients, whereas Schauder-type fixed point arguments can be applied to non-Lipschitz equations. The approach works equally well for multidimensional as for one-dimensional equations and leads to results in several interesting cases such as equations with path-dependent coefficients, anticipating equations, McKean-Vlasov type equations and equations with coefficients of superlinear growth.

Lluís Quer-Sardanyons, Universitat Autnoma Barcelona

Existence of Density for the Stochastic Wave Equation with Space-time Homogeneous Noise

We consider the stochastic wave equation in spatial dimension one or two, driven by a linear multiplicative space-time homogeneous Gaussian noise whose temporal and spatial covariance structure are given by locally integrable functions, which are the Fourier transforms of tempered measures. The main result shows that the law of the solution of this equation is absolutely continuous with respect to the Lebesgue measure, provided that the spatial spectral measure satisfies an integrability condition which ensures that the sample paths of the solution are Hölder continuous.

Xiaoming Song, Drexel University

Probability Density of Lognormal Fractional SABR Model

Instantaneous volatility of logarithmic return in the lognormal fractional SABR model is driven by the exponentiation of a correlated fractional Brownian motion. Due to the mixed nature of driving Brownian and fractional Brownian motions, probability density for such a model is less studied in the literature. We show in this paper a bridge representation for the joint density of the lognormal fractional SABR model in a Fourier space. Evaluating the bridge representation along a properly chosen deterministic path yields a small time asymptotic expansion to the leading order for the probability density of the fractional SABR model. A direct generalization of the representation to joint density at multiple times leads to a heuristic derivation of the large deviations principle for the joint density in small time. Approximation of implied volatility is readily obtained by applying the Laplace asymptotic formula to the call or put prices and comparing coefficients.
This is a joint work with Jiro Akahori and Tai-Ho Wang.

Samy Tindel, Purdue University

Discrete Rough Paths and Limit Theorems

In this talk we focus on a series of results concerning p-variation limits, as well as Itô type formulas in law for Gaussian processes. This line of research has been quite active in the recent past in the stochastic analysis community. Most of the techniques involve integration by parts, Stein's method, and other Malliavin calculus tools. This yields a series of limitations on the nature of the results, as well as the dimension of the Gaussian process at stake. Our aim is to show how those questions can possibly be handled in a more natural way thanks to rough path type techniques. More specifically we will show how to transfer limits taken on a Gaussian signature to limits involving controlled processes, by means of the typical expansions of the rough paths theory. Applications of this rather simple trick include the aforementioned p-variations and Itô type formulas, as well as central limit theorems for numerical schemes.

Ciprian Tudor, Universit Paris 1

Correlation Structure and Quadratic Variations of the Solution to the Wave Equation

We consider the stochastic wave equation with white or fractional noise in time and white noise in spce. We analyze the correlation structure of this Gaussian process and to study the asymptotic behavior in distribution of its spatial quadratic variation by using the tools from Malliavin calculus. As an application, we construct a consistent estimator for the Hurst parameter.

Tai-Ho Wang, Barach College, City University of New York

Target Volatility Option Pricing in Lognormal Fractional SABR Model

We examine the pricing of target volatility options in the lognormal fractional SABR model. A decomposition formula by Ito's calculus yields a theoretical replicating strategy for the target volatility option, assuming the accessibility of all variance swaps and swaptions. The same formula also suggests an approximation formula for the price of target volatility option in small time by the technique of freezing the coefficient. Alternatively, we also derive closed formed expressions for a small volatility of volatility expansion of the price of target volatility option. Numerical experiments show accuracy of the approximations in a reasonably wide range of parameters.

Jing Wu, Sun Yat-Sen University

Limit Theorems for Multivalued Stochastic Differential Equations

In this work, we consider the Euler and Yosida approximations of multivalued stochastic differential equations with non-Lipschitz coefficients and establish various limit theorem results.

Panqiu Xia, University of Kansas

Joint Holder Continuity of the Solutions to a Class of SPDEs Arising from Multi-dimensional Superprocesses in the Random Environment

We consider a d-dimensional branching particle system in a random environment. Suppose the initial measure converges to a finite maesure, which has a bounded Lebesgue density. Under some particular branching mechanism, we prove the equipped empirical measure converges almost surely to a finite measure-valued process in the weak topology and the limit process has a Lebesgue density. The density is also a weak solution to an SPDE. By using the techniques of Malliavin calculus and a conditional convolution representation, we prove that the density is jointly Holder continuous with time exponent less than 1/2, and spatial exponent less than 1.

George Yuan , Soochow University & BBD Inc.

The Dynamics of Stochastic Incentive Effect for "U" Shape Theory for SMEs under Bigdata Framework

In this paper, based on Higgs which is our Hologram engine under the bigdata, we establish the Dynamics of Stochastic Incentive Effect for "U" Shape Theory owned by SMEs (Small and medium-sized enterprises) by applying the stochastic resonance through the establishing nonlinear SDE to describe the dynamic behaviors of bilateral partnership system.
The aim of our talk is to discuss a new quantitative method and the associated prototype system to address the issue how the venture capital incents partners especially associated with partnership success, what roles the internal/external risks play respectively, and by how to avoid risk resonance and create portfolio strategies of introducing venture capital and optimizing the portfolio risk in the practice.
In another way, if taking the enterprise as the target (or say the partners), we like to describe the mechanics for venture capital finance in an environment by combining investment associated external and internal risk with consideration of capital-product switching mechanics - the "back and forth conversion of two states" (which are due to partnerships between multiple sides that share goals and strive for mutual benefit are ubiquitous both between and within the enterprises, and competition and cooperation are the fundamental characterize in partnership systems). In order to do so, we use "asymmetric bistable Cobb-Douglas utility" as the tool to describe the two states (actually we can also use some other kinds of utility function, too), then we build the new model called "nonlinear stochastic differential equation to describe the dynamical behaviors of bilateral partnership system in the presence of periodic capital-product switches and stochastic fluctuations" (called "an over-damped non- linear Langevin equation") to study when the back and forth conversion of two state" could reach the "best" in terms of Stochastic Resonance (SR) by introducing three new concepts below for the measurements of the system (for enterprise):
1) "output signal-to-noise ratio (SNR)",
2) "stationary unit risk-return (URR)", and
3) "incentive risk". These three new concepts can be classified as two categories: systematic risk, and bilateral risk.
In this way, we are able to establish the general framework for the mechanics of enterprises, in particular, to successfully explain the so-called "U" phenomenon for SMEs, the key business behavior of SMEs (from the practice in China) which mean that more external cooperators many not be better, this is against the intuition and traditional understanding (this might be one of biggest discovery for the SMEs study under the framework of Fintech by using bigdata method), which is also called "U" Shape phenomenon for SMEs first time in this area.

Jianfeng Zhang, University of Southern California

A Martingale Approach for Fractional Brownian Motions and Related Path Dependent PDEs

Motivated by option pricing in a financial market with rough volatility, we study backward SDEs in a framework where the (forward) state process satisfies a Volterra type SDE, with fractional Brownian motion as a typical example. Such processes are neither Markov processes nor semimartingales, and most notably, they feature a certain time inconsistency which makes any direct application of Markovian ideas, such as flow properties, impossible without passing to a path-dependent framework. Our main result is a functional Itô formula, extending the seminal work of Dupire to our more general framework. In particular, unlike in Dupire's setting where one needs only to consider the stopped paths, here we need to concatenate the observed path up to the current time with a certain smooth observable curve derived from the distribution of the future paths. This new feature is due to the time inconsistency involved in this paper. We then derive the path dependent PDEs for the backward problems. The talk is based on a joint work with Frederi Viens.

Xicheng Zhang, Wuhan University

Dirichlet Problem for Supercritical Non-local Operators

Let D be a bounded C^2-domain. Consider the following Dirichlet initial-boundary problem of nonlocal operators with a drift:

$$\partial_t u = \mathscr{L}_\kappa^{(\alpha)} u + b \cdot \nabla u + f \text{ in } \mathbb{R}_+ \times D, \quad u|_{\mathbb{R}_+ \times D^c} = 0, \ u(0, \cdot)|_D = \varphi,$$

where $\alpha \in (0,2)$ and $\mathscr{L}_\kappa^{(\alpha)}$ is an α-stable-like nonlocal operator with kernel function $\kappa(x,z)$ bounded from above and below by positive constants, and $b : \mathbb{R}^d \to \mathbb{R}^d$ is a bounded C^β-function with $\alpha + \beta > 1$, $f : \mathbb{R}_+ \times D \to \mathbb{R}$ is a C^γ-function in D uniformly in t with $\gamma \in ((1-\alpha) \vee 0, \beta]$, $\varphi \in C^{\alpha+\gamma}(D)$. Under some Hölder assumptions on κ, we show the existence of a unique classical solution $u \in L^\infty_{loc}(\mathbb{R}_+; C^{\alpha+\gamma}_{loc}(D)) \times C(\mathbb{R}_+; C_b(D))$ to the above problem. When $\alpha \in [1,2)$, the kernel κ can be rough in z. Moreover, we also establish the following probabilistic representation for u

$$u(t,x) = \mathbb{E}_x\Big(\varphi(X_t)\mathbf{1}_{\tau_D > t}\Big) + \mathbb{E}_x\left(\int_0^{t\wedge\tau_D} f(t-s, X_s)\mathrm{d}s\right), \ t \geq 0, \ x \in D,$$

where $((X_t)_{t\geq 0}, \mathbb{P}_x; x \in \mathbb{R}^d)$ is the Markov process associated with the operator $\mathscr{L}_\kappa^{(\alpha)} + b \cdot \nabla$, and τ_D is the first exit time of X from D. (This is a joint work with Guohuan Zhao.)