Yanpeng Li, University of Geneva, Switzerland

Poisson Geometry, Crystals, and Integrable Systems

For the Poisson-Lie dual K^* of a compact semisimple Lie group K, we construct a Poisson manifold $\mathrm{PT}(K^*) = \mathcal{C} \times T$ with a constant Poisson structure (here $\mathcal{C}$ is a certain polyhedral cone and T is a torus). The manifold $\mathrm{PT}(K^*)$ carries natural completely integrable systems with action-angle variables. In the case of $K = SU(n)$, one of these integrable systems is the Gelfand-Cetlin completely integrable system. Our main result is a one-to-one correspondence between generic symplectic leaves in $\mathrm{PT}(K^*)$ and generic coadjoint orbits in $\mathrm{Lie}(K)^*$ preserving symplectic volumes of the leaves. This observation gives hope to construct a dense Darboux chart in $\mathrm{Lie}(K)^*$ modeled on $\mathrm{PT}(K^*)$.

Our proof is based on the comparison of the Poisson-Lie dual G^* of $G = K^{\mathbb{C}}$ and of its Langlands dual $G^\vee$. We show that the integral cone defined by the cluster structure and the Berenstein-Kazhdan potential on the double Bruhat cell $G^{\vee; w_0, e} \subset G^\vee$ is isomorphic to the integral Bohr-Sommerfeld cone defined by the Poisson structure on $\mathrm{PT}(K^*)$. This statement implies the equality of volumes of symplectic leaves.

This is a joint work with A. Alekseev, A. Berenstein, B. Hoffman, and J. Lane.