



# Frontiers of Mathematics Lecture

## Potential Singularity Formation of the 3D Euler Equations and Related Models

### Professor Thomas Y. Hou

*Charles Lee Powell Professor of Applied & Computational Mathematics  
Department of Computing and Mathematical Sciences  
California Institute of Technology, USA*

**Date** : October 23, 2018 (Tuesday)

**Time** : 5:30 – 6:30 pm

*(Tea Reception starts at 5:00 pm)*

**Venue** : Lecture Theatre A, G/F, Chow Yei Ching Building,  
The University of Hong Kong



### Abstract

Whether the 3D incompressible Euler equations can develop a singularity in finite time from smooth initial data is one of the most challenging problems in mathematical fluid dynamics. This question is closely related to the Clay Millennium Problem on 3D Navier-Stokes Equations. A potential singularity in the 3D Euler equations is significant because it may be responsible for the onset of energy cascade in turbulent flows. We first review some recent theoretical and computational studies of the 3D Euler equations. Our study suggests that the convection term could have a nonlinear stabilizing effect for certain flow geometry. We then present strong numerical evidence that the 3D Euler equations develop finite time singularities. The singularity is a ring like singularity that occurs at a stagnation point in the symmetry plane located at the boundary of the cylinder. A careful local analysis also suggests that the blowing-up solution is highly anisotropic and is not of Leray type. A 1D model is proposed to study the mechanism of the finite time singularity. We have recently proved rigorously that the 1D model develops finite time singularity. Finally, we present some recent progress in developing an integrated analysis and computation strategy to analyze the finite time singularity of the original 3D Euler equations. We will also discuss possible implication of the 3D Euler singularities on the 3D Navier-Stokes equations.

### Biography

Thomas Yizhao Hou is the Charles Lee Powell Professor of applied and computational mathematics at Caltech. His research interests include 3D Euler singularity, interfacial flows, multiscale problems, and adaptive data analysis. He received his Ph.D. from UCLA in 1987, and joined the Courant Institute as a postdoc in 1987. He became a tenure track assistant professor at the Courant Institute in 1989 and then was promoted to tenured associate professor in 1992. He moved to Caltech in 1993 as a tenured associate professor and was named the Charles Lee Powell Professor in 2004. Dr. Hou has received a number of honors and awards, including Fellow of American Academy of Arts and Sciences in 2011, a member of the inaugural class of SIAM Fellows in 2009 and AMS Fellows in 2012, the Computational and Applied Sciences Award from USACM in 2005, the Morningside Gold Medal in Applied Mathematics in 2004, the SIAM Wilkinson Prize in Numerical Analysis and Scientific Computing in 2001, the Frenkiel Award from the Division of Fluid Mechanics of American Physical Society in 1998, the Feng Kang Prize in Scientific Computing in 1997, a Sloan fellow from 1990 to 1992. He was also the founding Editor-in-Chief of the SIAM Journal on Multiscale Modeling and Simulation from 2002 to 2007.