THE UNIVERSITY OF HONG KONG

Institute of Mathematical Research
Department of Mathematics

Number Theory Seminar

Arithmetic Statistics for Quaternion Algebras

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Abstract

Let F be a number field of degree d over $\mathbb{Q}$. We consider a quaternion algebra B over F, and write R for the places of F where B is not split. We introduce $G = Z\backslash B^\times$ the group of its projective units, where Z denotes the centre of $B^\times$. Let $\mathcal{A}(G)$ denote the set of all irreducible automorphic infinite dimensional representations of $G(\mathbb{A})$, taken up to isomorphism. A deep understanding of $\mathcal{A}(G)$, called the universal family of G following Sarnak, is of fundamental importance in the theory of automorphic forms.

We introduce a suitable notion of size on automorphic representations : the analytic conductor, that encapsulates the complexity of π and allows us to truncate the universal family of PGL_2, and hence that of G, to a finite set:

$$\mathcal{A}(Q) = \{\pi \in \mathcal{A}(G) \ : \ c(\pi) \leqslant Q\}.$$

Based on the Selberg trace formula and on a fundamental geometric interpretation of the analytic conductor, we establish the first arithmetic statistics results on this universal family : Weyl law, Plancherel equidistribution, Sato-Tate conjectures, type of symmetry of the low-lying zeros of the associated L-functions, etc. We will essentially focus on the arguments necessary to establish the following counting law :

For any $Q \geqslant 1$, we have

$$|\mathcal{A}(Q)| = CQ^2 + \begin{cases} O\left(Q^{1+\varepsilon}Q\right) & \text{if } F = \mathbb{Q} \text{ and } B \text{ totally definite;} \\ O\left(Q^{2-\delta_F}\right) & \text{if } F \neq \mathbb{Q} \text{ and } B \text{ totally definite;} \\ O\left(\frac{Q^2}{\log Q}\right) & \text{if } B \text{ is not totally definite.} \end{cases}$$

Date:	November 5, 2019 (Tuesday)
Time:	2:00 – 3:00pm
Venue:	Room 210, Run Run Shaw Bldg., HKU

All are welcome