Volume computation, optimisation, and comparing hierarchies of upper and lower bounds

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Abstract

In this talk we first introduce the pushforward measure associated with some mapping and provide two applications:

(i) Global optimization of polynomials on a simple set “K” like a box, an ellipsoid, a simplex, a hypercube or its image by an affine transformation.
(ii) Computing the Lebesgue volume of a basic semi-algebraic set.

By this approach one reduces a difficult multivariate problem to a simpler univariate one. The difficulty has now been transferred into computing the data of the (easier to solve) univariate problem. This in turn reduces to computing Lebesgue integrals of powers of some given polynomial on the simple set “K”. Although it is a fast and easy operation for modest size dimension, it can become tedious when the dimension and the power increase. This raises a new challenging problem about polynomial integration on simple sets. Finally, if time permits we will also introduce a common framework to describe and compare the Moment-SOS-hierarchies of upper and lower bounds in polynomial optimization.

Biography:

Jean-Bernard Lasserre graduated from “École Nationale Supérieure d’Informatique et Mathématiques Appliquées” (ENSIMAG) in Grenoble, then got his PhD (1978) and “Doctorat d’État” (1984) degrees both from Paul Sabatier University in Toulouse (France). He has been at LAAS-CNRS in Toulouse since 1980, where he is currently Directeur de Recherche (emeritus). He is also a member of IMT, the Institute of Mathematics of Toulouse, and holds the “Polynomial Optimization” chair at the ANITI Institute (one of the four recently created Artificial Intelligence Institutes in France). His past and present research activities cover machine Learning, applied mathematics, control and non-linear PDEs, probability, Markov control processes, approximation theory & convex optimization, production planning & scheduling. In particular, he has initiated the “Moment-SOS hierarchy” also known as “Lasserre hierarchy”, a novel methodology used in many areas for solving hard nonconvex polynomial optimization problems. He is SIAM Fellow (class 2014) and the laureate of the 2015 John von Neumann Theory prize of the INFORMS society, 2015 Khachiyan prize of the Optimization Society of INFORMS and 2009 Lagrange prize in Continuous optimization (awarded jointly every 3 years by SIAM and the Mathematical Optimization Society). He is invited speaker at the International Congress of Mathematicians in 2018.