



# Numerical Analysis Seminar

## Can Algorithms Collaborate? The Replica Exchange Method and Its Spectral Gap

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### Abstract

Gradient descent (GD) is known to converge quickly for convex objective functions, but it can be trapped at local minima. On the other hand, Langevin dynamics (LD) can explore the state space and find global minima, but in order to give accurate estimates, LD needs to run with a small discretization step size and weak stochastic force, which in general slows down its convergence. This talk shows that these two algorithms can “collaborate” through a simple exchange mechanism, in which they swap their current positions if LD yields a lower objective function. This idea can be seen as the singular limit of the replica-exchange technique from the sampling literature. We show that this new algorithm converges to the global minimum linearly with high probability, assuming the objective function is strongly convex in a neighborhood of the unique global minimum. By replacing gradients with stochastic gradients, and adding a proper threshold to the exchange mechanism, our algorithm can also be used in online settings. We further verify our theoretical results through some numerical experiments and observe superior performance of the proposed algorithm over running GD or LD alone. We will further explain how does replica exchange method improve LD’s spectral gap using Poincare type of inequality.

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| Date:  | April 6, 2022 (Wednesday)  |
| Time:  | 2:00 – 3:00pm (Hong Kong Time)   |
| Venue: | ZOOM: <a href="https://hku.zoom.us/j/">https://hku.zoom.us/j/</a><br>Meeting ID: 913 6532 3891<br>Password: 310656 |