THE UNIVERSITY  OF HONG KONG

Institute of Mathematical Research
Department of Mathematics

Number Theory Seminar

On the Universal Sums of Generalized Heptagonal Numbers

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Abstract

For a given $m \in \mathbb{N}$ with $m \geq 3$ and $x \in \mathbb{Z}$, the *x-th generalized m-gonal number* is denoted by

$$p_m(x) := \frac{(m-2)x^2 - (m-4)x}{2}. \tag{1}$$

Particularly, a generalized heptagonal number is of the form, $p_7(x) = \frac{5x^2-3x}{2}$. A sum of such generalized m-gonal numbers is given by

$$n = \sum_{j=1}^{l} a_j p_m(x_j), \tag{2}$$

where $\mathbf{a} \in \mathbb{N}^l$. Such a sum is considered universal for a given choice of $\mathbf{a}$, if the sum is solvable for all positive integers n. Specifically, we are interested in determining whether a sum is universal or not, given the choice of $\mathbf{a}$, for $m = 7$.

We define γ_m to be the smallest positive integer such that a sum of generalized m-gonal numbers is universal if and only if it represents all positive integers up to γ_m. Bosma and Kane established $\gamma_6 = \gamma_3 = 8$. Conway-Schneeberger 15 theorem implies $\gamma_4 = 15$ and Ju proved that $\gamma_5 = 109$.

In this joint collaborative project with Prof. Tomiyasu of Kyushu University and Prof. Kane of HKU, we use modular forms theory to get an explicit upper bound for γ_7. In particular, our main theorem in this project is that $\gamma_7 \leq 3.896 \cdot 10^{106}$. Although, based on the data obtained, we believe that γ_7 is as small as 131.

Date:	April 4, 2022 (Monday)
Time:	2:00 – 3:00pm (Hong Kong Time)
Venue:	ZOOM: https://hku.zoom.us/j/
	Meeting ID: 232 576 6007

All are welcome