



The Hong Kong Mathematical Society

(Founded in 1979)

Tel.: (852) 3943 8036 Fax: (852) 2603 7636 URL: http://www.hkms.org The Hong Kong Mathematical Society The Institute of Mathematical Sciences Unit 601, Academic Building No.1 The Chinese University of Hong Kong Shatin, N.T., Hong Kong

THE HONG KONG MATHEMATICAL SOCIETY ANNUAL GENERAL MEETING 2015

23 May 2015 (Saturday) 9:00am- 6:30pm

Lady Shaw Building The Chinese University of Hong Kong

Schedule of Events

Venue: LSB LT1

Chair: Zhouping Xin

9:00am - 10:00am HKMS Distinguished Lecture by Sijue Wu (University of Michigan)

<u>Title</u>: Mathematical Analysis of the Motion of Water Waves

- <u>Abstract</u>: In this talk, I will present the recent understandings on the motion of water waves obtained via rigorous mathematical tools, this includes the evolution of smooth initial data and some typical singular behaviors. We will also discuss some related open problems.
- 10:00am 10:15am Coffee Break

Chair: Ronald Lui

10:15am - 10:30am HKMS Best Thesis Award Presentation Ceremony

Chair: Conan Leung

10:30am - 11:20am Plenary Lecture 1 by Huai-Dong Cao (Lehigh University)

Title: Recent Developments of Ricci Solitons

<u>Abstract</u>: The notion of Ricci solitons was introduced by R. Hamilton in the mid-1980s to model singularity formations in the Ricci flow. Ricci solitons are self-similar solutions of the Ricci flow, or fixed points of the Ricci flow (considered as a dynamic system) modulo diffeomorphisms and scalings. They are also natural extensions of Einstein metrics. In this talk, we shall survey some recent developments of gradient Ricci solitons and discuss certain open problems.

Venue: LBS LT1

Chair: Zhouping Xin 11:25am - 11:55pm	HKMS Member Meeting
12:00pm - 2:00pm	Lunch at CC staff canteen (See the map) All faculties and invited speakers are welcome to join. We also have the student canteen at Franklin Center (See the map)

Venue: LSB LT1

Chair: Xiaoping Wang

2:00pm – 2:50pm Plenary Lecture 2 by <u>Yang Wang</u> (Hong Kong University of Science and Technology)

Title: Can Mathematics Solve Mysteries in Literature?

<u>Abstract</u>: There have been no shortages of controversies in literature, from old questions such as whether Cao Xueqin wrote all 120 chapters of "Dreams of Red Chamber", widely known as the greatest work of Chinese literature, to new questions such as whether Obama actually wrote his autobiography "Dreams From My Father". For mathematicians, it is interesting to ask whether mathematics can be used to settle these controversies. In this talk, I will give an overview on how mathematics can be applied to analyze the "style" of an author and the related field of study called "stylometry". I will show that mathematics can be used to almost definitively settle many such controversies.

2:50pm – 3:15pm Coffee Break

Invited talks (Parallel Sessions):

Venue: LSB LT1 Parallel Session 1: PDE Chair: Renjun Duan

3:15pm - 3:45pm 1. Xueke Pu (Chongqing University)

Title: Some Limit Problems of the Euler-Poisson System

<u>Abstract</u>: In this talk, I will discuss some limit problems for the Euler-Poisson system. In particular, I will discuss the long wavelength limit which leads to the classical KdV equation in 1D.

3:45pm - 4:15pm 2. <u>Wei Xiang</u> (City University of Hong Kong)

<u>Title</u>: Shock Diffraction by Convex Cornered Wedges

<u>Abstract</u>: In this talk, I would like to present one of our current research projects, that is on the mathematical analysis of shock diffraction by convex cornered wedges for the potential flow. The existence of the regular configuration is established for any cornered wedge when the upcoming shocks are weak.

4:15pm - 4:45pm **3.** Zhian Wang (Hong Kong Polytechnic University)

Title: Chemotactic Waves and Their Applications

<u>Abstract</u>: Compared to the classical chemotaxis models with linear chemotactic sensitivity, logarithmic sensitivity has more specific applications in modeling biological processes such as the bacterial movement and onset of tumor angiogenesis. The typical patterns generated by the logarithmic sensitivity are traveling waves which were explicitly observed in the experiment. However the logarithmic sensitivity induces a singularity and hence brings a great challenge for mathematical analysis. In this talk, the recent progress of existence and stability of traveling waves of chemotaxis model with logarithmic sensitivity will be reported and some new insights and open questions will be discussed.

4:45pm -5:15pm 4. Xiangpeng Hu (City University of Hong Kong)

Title: Weak Solution of Incompressible Viscoelasticity

<u>Abstract</u>: In this talk, the weak formulation of incompressible viscoelasticity is formulated and the global well-posedness near the equilibrium in the energy space will be the focus.

Venue: LSB LT2

Parallel Session 2: Optimization Chair: Ting Kei Pong

3:15pm - 3:45pm 1. Yafeng Liu (Chinese Academy of Sciences)

<u>Title</u>: Composite $L_q(0 < q < 1)$ Minimization over Polyhedron

<u>Abstract</u>: The composite $L_q(0 < q < 1)$ minimization problem over a general polyhedron has received various applications in machine learning, wireless communications, image restoration, signal reconstruction, etc. In this talk, we shall present a theoretical study on this problem. Firstly, we show that for any fixed 0 < q < 1, finding the global minimizer of the problem, even its unconstrained counterpart, is strongly NP-hard. Secondly, we derive Karush-Kuhn-Tucker (KKT) optimality conditions for local minimizers of the problem. Thirdly, we propose a smoothing sequential quadratic programming framework for solving this problem. The framework requires a (approximate) solution of a convex quadratic program at each iteration. Finally, we analyze the worst-case iteration complexity of the framework for returning an ϵ –KKT point; i.e., a feasible point that satisfies a perturbed version of the derived KKT optimality conditions. To the best of our knowledge, the proposed framework is the first one with a worst-case iteration complexity guarantee for solving composite L_q minimization over a general polyhedron. This is a joint work with Shiqian Ma, Yu-Hong Dai, and Shuzhong Zhang.

3:45pm - 4:15pm 2. Zhaosong Lu (Simon Fraser University)

Title: Orthogonal Rank-One Matrix Pursuit for Matrix Completion

<u>Abstract</u>: In this talk, we consider low rank matrix completion problem, which has wide applications such as collaborative filtering, image inpainting and Microarray data imputation. In particular, we present an efficient and scalable algorithm for matrix completion. In each iteration, we pursue a rank-one matrix basis generated by the top singular vector pair of the current approximation residual and update the weights for all rank-one matrices obtained up to the current iteration. We further propose a novel weight updating rule to reduce the time and storage complexity, making the proposed algorithm scalable to large matrices. We establish a linear rate of convergence for the algorithm. Numerical experiments on many real-world large scale datasets demonstrate that our algorithm is much more efficient than the state-of-the-art algorithms while achieving similar or better prediction performance.

4:15pm - 4:45pm **3.** <u>Ting Kei Pong</u> (Hong Kong Polytechnic University)

Title: Douglas-Rachford Splitting for Nonconvex Feasibility Problems

<u>Abstract</u>: The Douglas-Rachford (DR) splitting method is a popular approach for finding a point in the intersection of two closed convex sets and has also been applied very successfully to various nonconvex instances, even though the theoretical justification in this latter setting is far from being complete. In this talk, we aim at understanding the behavior of the DR splitting in finding an intersection point of a closed convex set C and a possibly nonconvex closed set D. In particular, we adapt this method to minimize the square distance to C subject to D. When the stepsize is small and either C or D is compact, we show that the sequence generated is bounded and any cluster point gives a stationary point of the minimization problem. Moreover, if C and D are in addition semi-algebraic, then the whole sequence is convergent. We also discuss a generalization of the method to minimize the sum of a Lipschitz differentiable function and a proper closed function, both possibly nonconvex. We present preliminary numerical results comparing the DR splitting against the alternating projection method. This is joint work with Guoyin Li.

4:45pm -5:15pm 4. <u>Man-cho So, Anthony</u> (The Chinese University of Hong Kong)

<u>Title</u>: A Perturbation Inequality for Concave Functions of Singular Values and Its Applications in Low-Rank Matrix Recovery

<u>Abstract</u>: In this talk, we will present our recent progress on proving a perturbation inequality for concave functions of singular values, which allows us to answer an open question that is of interest to both the compressive sensing and linear algebra communities. As a result, we are able to confirm the validity of a number of previously conjectured conditions for the recovery of low-rank matrices via the popular Schatten p-quasi-norm heuristic. We shall also discuss some potential applications of our inequality in low-rank matrix recovery.

3:15pm - 3:45pm **1.** <u>Tsz Shun Chung, Eric</u> (The Chinese University of Hong Kong)

<u>Title</u>: A Multiscale Model Reduction Technique for the Wave Equation and Its Application to Inverse Problems

<u>Abstract</u>: In this talk, we present a new multiscale model reduction technique for the wave equation in heterogeneous media. The method is based on the generalized multiscale finite element method. The space reduction is obtained through the use of some local spectral problems. Our new method allows a rapid simulation of waves in highly heterogeneous media. We will also present the application of the method to a class of inverse problems.

3:45pm - 4:15pm **2.** Jingzhi Li (South University of Science and Technology of China)

<u>Title</u>: Locating the Buried Objects in a Two-layered Medium by a Single Far-field Measurement

<u>Abstract</u>: We develop an inverse scattering scheme of recovering anomalies buried in a twolayered medium. The recovery scheme works in a very general setting and possesses several salient features. It makes use of a single far-field measurement in the half-space above the anomalies, and works independently of the physical properties of the anomalies. There might be anomalous components of multiscale sizes presented simultaneously. Moreover, the proposed scheme is of a totally direct nature without any inversion involved, and hence it is very fast and robust against measurement noise. Both theoretical foundation and numerical experiments are presented. This extends related results in the literature on recovering multiscale scatterers located in homogeneous space. This is a joint work with Peijun Li, Hongyu Liu and Xiaodong Liu.

4:15pm - 4:45pm 3. <u>Hongyu Liu</u> (Hong Kong Baptist University)

Title: Stable Determination by a Minimal Number of Far-field Measurements

<u>Abstract</u>: In this talk, the speaker will present two stability results of logarithmic type in determining conducting scatterers by a minimal number of electromagnetic (EM) far-field measurements. The admissible Lipschitz scatterers are of general polyhedral type, which may consist of finitely many (with an unknown number) solid polyhedra and screen-type cells. They satisfy minimum regularity assumptions. For the general case, the stability estimate is established by using two EM far-field measurements. If there are no screen-type components presented, then the stability estimate is established by using only a single far-field measurement.

4:45pm -5:15pm **4.** Shingyu Leung (Hong Kong University of Science and Technology)

<u>Title</u>: Numerical Approaches for Eikonal-based Traveltime Tomography Problem

<u>Abstract</u>: The talk discusses two Eulerian formulations for solving the first arrival traveltime tomography arising from important applications such as seismic imaging and medical imaging. In the first approach, we define a mismatch functional and derive the gradient of the nonlinear functional by an adjoint state method. The new approach formulates the traveltime tomography problem as a variational problem of a certain cost functional explicitly with respect to both traveltime and sound speed. Furthermore, the cost functional is penalized to enforce the nonlinear equality constraint associated with the underlying eikonal equation, biharmonically regularized with respect to traveltime, and harmonically regularized with respect to sound speed. This resulting system is associated with an initial value problem which can be efficiently solved by an operator-splitting based solution method.

Venue: LSB LT4 Parallel Session 4: New Faculty Chair: Jintao Cui

3:15pm - 3:45pm 1. <u>Wing-Cheong Lo</u> (City University of Hong Kong)

Title: Pattern in a Cell: Modeling Cell Polarization in Budding Yeast

<u>Abstract</u>: Robust cell polarity is critical for cell survival and normal tissue development. Cell polarity is usually induced through the localization of specific molecules to a proper location of the cell membrane. Here we propose a generic model consisting the particle density of membrane bound molecules undergoing polarization to study the mechanisms for different budding patterns in yeast cells.

3:45pm - 4:15pm **2.** <u>Guo Xin</u> (Hong Kong Polytechnic University)

<u>Title</u>: The Local Edge Machine: Inference of Dynamic Models of Gene Regulation

<u>Abstract</u>: A gene regulatory network is a collection of genes that regulate each other, through RNA and protein expression products. Gene regulatory networks enable organisms to predict and adapt to environment changes. Understanding the regulatory relationship is a big challenge in molecular biology and systems biology. We developed the Local Edge Machine (LEM), which is an algorithm to infer the network from temporally dynamic gene expression data. LEM uses differential equation systems with the Hill function model to fit the expression level data, and is regularized with preference on robust systems. In validation studies on both *in silico* and *in vivo* data, our method outperforms previously reported methods by wide margins.

4:15pm - 4:45pm **3.** Jintao Cui (Hong Kong Polytechnic University)

Title: Multigrid Methods for Two-dimensional Maxwell's Equations on Graded Meshes

<u>Abstract</u>: In this work we investigate the numerical solution for two-dimensional Maxwell's equations on graded meshes. The approach is based on the Hodge decomposition. The solution u of Maxwell's equations is approximated by solving standard second order elliptic problems. The quasi-optimal error estimates for both u and curl of u in the L2 norm are obtained on graded meshes. We then prove the uniform convergence of the W-cycle and full multigrid algorithms for the resulting discrete problem. The performance of these methods is illustrated by numerical results.

4:45pm -5:15pm 4. Dennis Amelunxen (City University of Hong Kong)

Title: Theory and Applications of Conic Integral Geometry

- <u>Abstract</u>: The theory of (conic) integral geometry provides -exact- answers for questions about geometric probabilities, such as
 - the probability of nontrivial intersections of randomly oriented convex cones, or
 - the success probability of Compressed Sensing for Gaussian sensing matrices, or
 - the probability of having a solution of rank r for a random (GOE) semidefinite program.

Moreover, these answers can be simplified through concentration of measure results, which, as a special case, proves the existence of phase transitions in a class of convex approaches to data recovery problems, as well as simple tools for computing these. We will present an introduction to this theory of conic integral geometry and outline its diverse applications.

Venue: LSB LT5 Parallel Session 5: Best Thesis Awardees Chair: Ronald Lui

3:15pm - 3:45pm **1.** Zhen Zhang (National University of Singapore)

Title: The Phase Field Simulations for the Zone Melting Purification Method

<u>Abstract</u>: Zone melting has been used in the purification of industrial phosphorus recently. It is a process consisting of both the melting-solidification process and the solute diffusion process. We propose a variational phase-field model to simulate the zone melting process. The resulting coupled system of Allen-Cahn equation, heat equation and solute diffusion equation is solved numerically. To overcome the stiffness caused by the nonlinear term in Allen-Cahn equation, we propose a semi-implicit scheme based on a convex splitting of the free energy. By a proper splitting of the free energy functional, it can be shown that the semi-implicit scheme is unconditionally energy stable. Numerical experiments are carried out to verify the convergence and stability of our numerical scheme. Furthermore, we present our numerical simulations for the zone melting process and numerically show the feasibility of our model by comparing with experiments.

3:45pm - 4:15pm 2. <u>Yat-Ting Chow</u> (The Chinese University of Hong Kong)

<u>Title</u>: Some New Reconstruction Algorithms and Their Mathematical Theories for Inverse Medium Scattering Problems

<u>Abstract</u>: We are concerned with several inverse medium scattering problems. In this talk, we focus on the acoustic/transverse electric (TE)/transverse magnetic (TM) inverse scattering problems, and introduce a new notion of scattering coefficients for heterogeneous media. We analyse this new entity mathematically, and these analysis help us to obtain explicit reconstruction formulae in the linearized case. Based on this novel concept of scattering coefficients, sensitivity and resolution analysis are performed to mathematically assess the reconstruction quality and justify the super-resolution phenomenon in imaging high contrast targets. We will then move on to a very brief introduction to several other severely ill-posed inverse medium problems that we have also worked on, such as electric impedance tomography (EIT), diffusive optical tomography (DOT), and a shape design problem related to plasmon resonance. We give a very brief introduction to some simple and efficient new methods to solve these problems, e.g. the direct sampling methods (DSMs).

Invited talks (Student Sessions):

Venue: LSB LT2 Student Session 1 Chair: Chi Yeung Lam

5:30pm -5:45pm 1. Siu Hong Cheng (The Chinese University of Hong Kong)

Title: Monge-Ampère Equation and Optimal Transportation

<u>Abstract</u>: In this session, we briefly talk about the optimal transportation problem proposed by Monge, and show that it is related to a Monge-Ampère type equation

$$\det D^2(u(x)) = \frac{\rho(x)}{\rho^*(Du(x))} = f(x)$$

defined in a bounded domain Ω in \mathbb{R}^n . This equation is related to the optimal transportation problem associated with the quadratic cost $c(x, y) = x \cdot y$, or equivalently the distance squared cost $c(x, y) = \frac{1}{2}|x - y|^2$. We briefly sketch how to show the existence of solution to the second boundary value problem

$$\begin{cases} \det(D^2 u(x)) = \frac{\rho(x)}{\rho^* (Du(x))} & \text{in } \Omega\\ Du(\Omega) = \Omega^* & \text{on } \partial \Omega \end{cases}$$

where Ω , Ω^* are uniformly convex, which is closely related to the optimal transportation problem. Some techniques such as the maximum principle to derive the a priori estimates can be discussed if time allows.

5:45pm -6:00pm 2. Chi Yeung Lam (The Chinese University of Hong Kong)

<u>Title</u>: A Staggered Discontinuous Galerkin Method for the Simulation of Seismic Waves with Surface Topography

Abstract: Accurate simulation of seismic waves is of critical importance in a variety of geophysical applications. Based on recent works on staggered discontinuous Galerkin methods, we propose for the simulations of seismic waves a new method which has energy conservation and extremely low grid dispersion, so that it naturally provides accurate numerical simulations of wave propagation useful for geophysical applications and is a generalization of classical staggered grid finite difference methods. Moreover, it can handle with ease irregular surface topography and discontinuities in the subsurface models. Our new method discretizes the velocity and the stress tensor on this staggered grid, with continuity imposed on different parts of the mesh. The symmetry of the stress tensor is enforced by the Lagrange multiplier technique. The resulting method is an explicit scheme, requiring the solutions of a block diagonal system and a local saddle point system in each time step, and is therefore very efficient. In order to tailor our scheme to Rayleigh waves, we develop a mortar formulation of our method. In particular, a fine mesh is used near the free surface and a coarse mesh is used in the rest of the domain. The two meshes are in general not matching, and the continuity of the velocity at the interface is enforced by a Lagrange multiplier. The resulting method is also efficient in time marching. We also give a stability analysis of the scheme and an explicit bound for the time step size. In addition, we present some numerical results to show that our method is able to preserve the wave energy and accurately compute the Rayleigh waves. Moreover, the mortar formulation gives a significant speed up compared with the use of a uniform fine mesh, and provides an efficient tool for the simulation of Rayleigh waves.

6:00pm -6:15pm 3. Pui Tung Choi (The Chinese University of Hong Kong)

<u>Title</u>: Fast Disk Conformal Parameterization of Simply-connected Open Surfaces

<u>Abstract</u>: Surface parameterizations have been widely used in computer graphics and geometry processing. In particular, as simply-connected open surfaces are conformally equivalent to the unit disk, it is desirable to compute the disk conformal parameterizations of the surfaces. In this work, we propose a novel algorithm for the conformal parameterization of a simply-connected open surface onto the unit disk, which significantly speeds up the computation, enhances the conformality and stability, and guarantees the bijectivity. The conformality distortions at the inner region and on the boundary are corrected by two steps, with the aid of an iterative scheme using quasi-conformal theories. Experimental results demonstrate the effectiveness of our proposed method.

6:15pm -6:30pm 4. Dongfang Yun (City University of Hong Kong)

<u>Title</u>: The Localized Radial Basis Function Collocation Method for Numerical Simulation of Partial Differential Equations

<u>Abstract</u>: In this talk, a novel meshless method is introduced for numerical simulation of partial differential equations (PDEs). Compared with vastly and widely applied finite difference method and finite element method, the meshless method enjoys the advantage of easy implementation when solving multi-dimensional problems defined in complex geometric domains. Besides, the localized radial basis function collocation method (LRBFCM) is capable of dealing with large-scale problems with high accuracy. Solvability and stability have been proved numerically. To verify this, a benchmark thermo-driven fluid-flow problem is firstly tested. By LRBFCM, a system of coupled mass, momentum and energy conservation equations are solved numerically. Moreover, due to the localization property, different selections of local influence domain affect numerical results. Combine with upwind technique, LRBFCM is applied to solve one- and two-dimensional convection-dominated problems. Accurate and oscillation-free solutions are obtained in spite of boundary layer and shock wave phenomenon.

Venue: LSB LT3 Student Session 2 Chair: Lei Yang

5:30pm -5:45pm 1. <u>Qivu Wang</u> (Hong Kong Polytechnic University)

<u>Title</u>: Sparse Markowitz Portfolio Selection by Using Stochastic Linear Complementarity Approach

<u>Abstract</u>: We consider the framework of the classical Markowitz mean-variance model when multiple solutions exist, among which the sparse solutions are stable and cost-efficient. We propose a two-phase stochastic linear complementarity approach. This novel method stabilizes the optimization problem, finds the sparse asset allocation that saves the transaction cost, and is in the solution set of the Markowitz problem which results in minimum variance portfolio. Our approach could be applied to non-short-positions portfolios and short-position-allowed portfolios. Moreover, we apply the sample average approximation (SAA) method to the two-phase optimization problems and give detailed convergence analysis. We implement this methodology on the data sets constructed by Fama and French and real data in the newly launched Shanghai-Hong Kong Stock Connect scheme. With mock investment in training data, we construct portfolios, test them in the out-of-sample data and find their performance, measured by Sharpe ratio, VaR and CVaR, is superb compared with the 1/N strategy, and L_p regularized portfolios.

5:45pm -6:00pm 2. <u>Haibin Chen</u> (Hong Kong Polytechnic University)

Title: Sum-of-Squares Tensors and their Sum-of-Squares Rank

Abstract: A fundamental and challenging problem in dealing with tensors is to determine its positive semi-definiteness which is known to be an NP-hard problem in general. An important class of tractable positive semi-definite tensors is the sum-of-squares (SOS) tensors. SOS tensors have a close connection with SOS polynomials, which are very important in polynomial theory and polynomial optimization. In this paper, we examine SOS properties of classes of structured tensors, and study the SOS-rank of SOS tensors. We first establish SOS properties of various even order symmetric structured tensors available in the literature. These include weakly diagonally dominated tensors, B₀-tensors, double Btensors, quasi-double B_0 -tensors, MB_0 -tensors, H-tensors, and absolute tensors of positive semi-definite Z-tensors. We also examine the SOS-rank for SOS tensors and the SOS-width for SOS tensor cones. The SOS-rank provides the minimal number of squares in the sums-of-squares decomposition for the SOS tensors, and, for a specific SOS tensor cone, its SOS-width is the maximum possible SOS-rank for all the tensors in this cone. We first deduce an up bound for general SOS tensors and the SOS-width for general SOS tensor cone using the known results in the literature of polynomial theory. Then, we provide an explicit sharper estimate for SOS-rank of SOS tensors with bounded exponent and identify the SOS-width for the tensor cone consisting of all SOS tensors with bounded exponent. Finally, we also show that the SOS-rank of an SOS tensor is equal to the optimal value of a related rank optimization problem over positive semi-definite matrix constraint.

6:00pm -6:15pm 3. Lei Yang (Hong Kong Polytechnic University)

<u>Title</u>: Alternating Direction Method of Multipliers for Nonconvex Background/Foreground Extraction

<u>Abstract</u>: One important problem in video processing is the background/foreground extraction. In this talk, we propose a new optimization model to handle this problem. Our model is nuclear norm free, and can incorporate different possibly nonconvex sparsity inducing regularization functions for extracting the foreground, such as the L_p quasi-norm for $0 . To solve the resulting possibly nonconvex optimization problem, we adapt the alternating direction method of multipliers (ADMM) with general dual step-size. We establish its convergence under mild conditions. Finally, we present numerical experiments comparing our model solved by our ADMM against the <math>L_1$ -based model on both synthetic and real data. The preliminary numerical results demonstrate the efficiency of our model and algorithm.

6:15pm -6:30pm 4. <u>Hei Long Chan, Antonie</u> (The Chinese University of Hong Kong)

Title: Detection of n-dimensional Shape Deformities using n-dimensional Quasi-conformal Maps

<u>Abstract</u>: Detecting deformities on objects is a typical topic in shape analysis, and has much applications such as abnormalities detection in medical imaging. While many algorithms are already well-established in 2-dimensional case when the object is indeed a surface, one that still perform well in the general n-dimensional case is still missing. It is our goal to complete this missing piece, by introducing an indicator in order to effectively distinguish between normal and abnormal deformities. The proposed framework is closely related to the classic 2-dimensional conformal geometry and quasi-conformal geometry. We model abnormal deformations by anisotropic deformations. Given any two objects of the same dimension, we define the "Anisotropic Indicator" to demonstrate the abnormalities in the deformation. Both global and local features about the abnormalities between the two objects can be tracked by analyzing the indicator. We tested the algorithm by detecting deformations on synthetic data and real data, and results show that our algorithm can detect deformations of different types and degrees.



Free School Bus Schedule

Bus route for 1a

Bus route for 1b



