Fourier Analysis in Fractal Geometry

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Abstract

Many fractal objects (function, measure or distribution) possess self-similar properties. For example, scaling functions in wavelet and Bernouilli convolution measures. The Fourier transform of such an object (in one-dimensional case) satisfies a recursive relation like

$$F(\xi) = f_1\left(\frac{\xi}{\beta}\right) F\left(\frac{\xi}{\beta}\right) + f_2\left(\frac{\xi}{\beta^2}\right) F\left(\frac{\xi}{\beta^2}\right) + \cdots + f_d\left(\frac{\xi}{\beta^d}\right) F\left(\frac{\xi}{\beta^d}\right)$$

where $d \geq 1$ is an integer, $\beta > 1$ is a real number (called scaling factor) and $f_1, \cdots, f_d$ are 1-periodic functions. We study asymptotic properties at the infinity of F.

The special case of Bernoulli convolution ($d = 1, f_1(x) = \cos 2\pi x$) has been studied since the 1930's (Jessen, Wintner, Erdös, Salem, Kahane, Garsia, Solomyak). For the general case, R. Strichartz et al had made numerical analysis in the beginning of 1990's. Later, rigorous results were obtained under the open set condition (Strichartz, Lau). Using transfer operator as tool, the case where β is an integer was well studied (Fan-Lau). It is more difficult when $\beta > 1$ is not integral, because there is no longer useful dynamical device.

Together with B. Saussol and J. Schmeling we attack this difficulty. We first convert the problem to that of matrix products of the form $M(\beta^n x) \cdots M(\beta x) M(x)$. Notice that $M(\beta^n x)$ is not stationary. We prove that for Pisot number $\beta > 1$ or for almost all $\beta > 1$ the Liapunov exponent exists for almost every x. We also prove that under certain condition, the topological pressure exists. We are looking for relations between the topological pressure and the Liapunov exponent.