

# **Workshop on Complex Geometry**

**August 2 - 4, 2011**

**Room 210, Run Run Shaw Bldg., HKU**

**Program and Abstracts**



**Institute of Mathematical Research  
The University of Hong Kong**

*Speakers:*

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Meng Chen	Fudan University, Shanghai, China
Yik-Man Chiang	HK University of Science & Technology, Hong Kong
Lawrence Ein	University of Illinois, Chicago, USA
Baohua Fu	Chinese Academy of Sciences, Beijing, China
Peter Greiner	University of Toronto, Canada
Ronald Lui	The Chinese University of Hong Kong, Hong Kong
Sui-Chung Ng	HKU, Hong Kong
Tuen-Wai Ng	HKU, Hong Kong
Mounir Nisse	Texas A&M University, USA
Yum-Tong Siu	Harvard University, USA
Wing-Keung To	National University of Singapore
Jonathan Tsai	HKU, Hong Kong
Siye Wu	HKU, Hong Kong
Sai-Kee Yeung	Purdue University, USA

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<b>Time / Date</b>	<b>Aug 2 (Tue)</b>	<b>Aug 3 (Wed)</b>	<b>Aug 4 (Thur)</b>
<b>9:30 – 10:30</b>	<b>Siu</b>	<b>Greiner</b>	<b>Ein</b>
<b>10:30 – 10:50</b>	<b>Tea Break</b>		
<b>10:50 – 11:50</b>	<b>Ng SC</b>	<b>To</b>	<b>Chen</b>
<b>Lunch Break</b>			
<b>14:00 – 15:00</b>	<b>Wu</b>	<b>Nisse</b>	<b>Fu</b>
<b>15:10 – 16:10</b>	<b>Yeung</b>	<b>Tsai</b>	<b>Chiang</b>
<b>16:10 – 16:30</b>	<b>Tea Break</b>		
<b>16:30 – 17:30</b>	<b>Lui</b>	<b>Ng TW</b>	

# PROGRAM

**August 2, 2011**  
**Tuesday**

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9:30 – 10:30      **Yum-Tong Siu**, Harvard University, USA  
*Multiplier ideal sheaves in PDE*

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*Tea Break*

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10:50 – 11:50      **Sui-Chung Ng**, HKU, Hong Kong  
*Rigidity of holomorphic mappings on flag domains on complex Grassmannians*

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*Lunch Break*

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14:00 – 15:00      **Siye Wu**, HKU, Hong Kong  
*Analytic torsion of twisted de Rham and Dolbeault complexes*

15:10 – 16:10      **Sai-Kee Yeung**, Purdue University, USA  
*Holomorphic one forms, rational and integral points on complex two ball quotients*

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*Tea Break*

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16:30 – 17:30      **Ronald Lui**, The Chinese University of Hong Kong, Hong Kong  
*Computational Conformal/Quasi-conformal Geometry and its applications*

**August 3, 2011**  
**Wednesday**

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9:30 – 10:30      **Peter Greiner**, University of Toronto, Canada  
*On heat kernels on spheres*

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*Tea Break*

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10:50 – 11:50      **Wing-Keung To**, National University of Singapore  
*Syzygies of compact complex hyperbolic manifolds*

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*Lunch Break*

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14:00 – 15:00      **Mounir Nisse**, Texas A&M University, USA  
*A necessary and sufficient condition on analytic subvarieties of the complex torus to be algebraic*

15:10 – 16:10      **Jonathan Tsai**, HKU, Hong Kong  
*The Loewner driving function of periodic curves*

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*Tea Break*

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16:30 – 17:30      **Tuen-Wai Ng**, HKU, Hong Kong  
*Chebyshev-Blaschke products*

**August 4, 2011**  
**Thursday**

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9:30 – 10:30      **Lawrence Ein**, University of Illinois, Chicago, USA  
*Syzygies of higher dimensional varieties*

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*Tea Break*

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10:50 – 11:50      **Meng Chen**, Fudan University, Shanghai, China  
*On threefolds of general type with small genus:  $g = 1/2$*

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*Lunch Break*

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14:00 – 15:00      **Baohua Fu**, Chinese Academy of Sciences, Beijing, China  
*Prolongations of the linear automorphism group of a projective variety  
and applications*

15:10 – 16:10      **Yik-Man Chiang**, HK University of Science & Technology, HK  
*Complex oscillation theory and special functions*

## Abstracts

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**Meng Chen**, Fudan University, Shanghai, China

*On threefolds of general type with small genus:  $g = 1/2$*

Due to recent development on explicit birational geometry in dimension 3, we can classify 3-folds of general type into about a dozen of classes by introducing the “numerical genus  $g$ ” (as a birational invariant). From the definition,  $g = 1/n$  ( $2 \leq n \leq 18$ ) or  $g \geq 2$  and in fact, the case  $g \geq 2$  is known up to some extent. In the first part of this new series, we study some explicit birational geometry of 3-folds with  $g = 1/2$ .

**Yik-Man Chiang**, HK University of Science & Technology, Hong Kong

*Complex oscillation theory and special functions*

Complex oscillation theory is a study of the value distribution aspect of entire function solutions of certain complex linear differential equations in the complex plane by Nevanlinna’s value distribution theory. We shall give a survey of this area and show how it is related to special function equations with four regular singular points on the Riemann sphere.

**Lawrence Ein**, University of Illinois, Chicago, USA

*Syzygies of higher dimensional varieties*

This is joint work with Rob Lazarsfeld. Let  $X$  be a smooth projective variety and  $A$  be an ample divisor on  $X$ . We embeds  $X$  into  $P^N$  by the complete linear system  $|dA|$ . We study the minimal free resolution of the coordinate ring of  $X$  in  $P^N$ , when  $d$  is sufficiently large.

**Baohua Fu**, Chinese Academy of Sciences, Beijing, China

*Prolongations of the linear automorphism group of a projective variety and applications*

I shall give the classification of smooth non-degenerate projective varieties with non-zero prolongations with a sketch of proof. Then several applications will be given. This is a joint work with Jun-Muk Hwang.

**Peter Greiner**, University of Toronto, Canada

*On heat kernels on spheres*

I shall discuss a geometric construction of the heat kernel for both elliptic and subelliptic Laplacians on spheres in complex  $n$ -space.

**Ronald Lui**, The Chinese University of Hong Kong, Hong Kong

*Computational Conformal/Quasi-conformal Geometry and its applications*

Conformal (C)/Quasi-conformal (QC) geometry has a long history in pure mathematics, and is an active field in both modern geometry and modern physics. Recently, with the rapid development of 3D digital scanning technology, the demand for effective geometric processing techniques is ever increasing. Computational conformal/quasi-conformal geometry plays an important role for this purpose, and has found important applications in different areas such as medical imaging and computer graphics.

In practice, geometric structures are usually represented discretely by triangulation meshes. In this talk, I will firstly describe how C/QC theories can be discretized onto discrete meshes. This gives a discrete analogue of C/QC geometry on meshes. Then, I will talk about how computational C/QC geometry can be practically applied to different applications such as computer graphics and medical imaging for disease analysis.

**Sui-Chung Ng**, HKU, Hong Kong

*Rigidity of holomorphic mappings on flag domains on complex Grassmannians*

Let  $G$  be a complex simple Lie group and  $P$  be a parabolic subgroup. A flag domain on the rational homogeneous space (flag manifold)  $G/P$  is an open orbit of a real form  $G_0$  of  $G$ . In this talk, we will look at certain cycle spaces of the  $SU(m, n)$ -type flag domains on complex Grassmannians. The cycles that we are interested in are some totally geodesic subgrassmannians. We are going to deduce from the structure of these cycle spaces some rigidity results of holomorphic mappings between Grassmannians. These include a local characterization of the automorphisms of the above flag domains which is analogous to the classical Alexander-Henkin-Tumanov theorem on irreducible bounded symmetric domains.

**Tuen-Wai Ng**, HKU, Hong Kong

*Chebyshev-Blaschke products*

In a recent joint work with Mingxi Wang, a version of Ritt's theory on the factorization of finite Blaschke products has been developed. In this Ritt's theory on the unit disk, a special class of finite Blaschke products has been introduced as the counterpart of Chebyshev polynomials in Ritt's theory for polynomials. These special finite Blaschke products are therefore called Chebyshev-Blaschke products. In this talk, I will explain the construction of them and also discuss some of their interesting properties.



**Mounir Nisse**, Texas A&M University, USA

*A necessary and sufficient condition on analytic subvarieties of the complex torus to be algebraic*

In this talk we deal with generic analytic subvarieties of the complex algebraic torus  $(\mathbb{C}^*)^n$ . We show that a generic  $k$ -dimensional analytic subvariety of the  $n$ -dimensional complex torus is algebraic if and only if its logarithmic limit set is a finite rational complex polyhedron of dimension  $k - 1$ . This is equivalent to saying that its phase limit set contains no real torus of dimension strictly greater than  $k$ .

In particular, if the dimension of the ambient space,  $n$ , is at least  $2k$ , then, the last conditions are equivalent to the fact that the volume of the amoeba is finite. It is a stronger, tropical version of Chow's theorem.

**Yum-Tong Siu**, Harvard University, USA

*Multiplier ideal sheaves in PDE*

The theory of multiplier ideal sheaves was originally developed for problems of  $\bar{\partial}$ -estimates and regularity. In this talk we discuss the recent techniques of the theory and explore its applications to general systems of PDE and other problems in complex geometry.

**Wing-Keung To**, National University of Singapore

*Syzygies of compact complex hyperbolic manifolds*

In this talk, I will discuss some joint works with J.-M. Hwang. In particular, I will report on a recent joint work on the syzygies of the canonical bundle of compact complex hyperbolic manifolds.

**Jonathan Tsai**, HKU, Hong Kong

*The Loewner driving function of periodic curves*

The classical work of Charles Loewner allows us to represent a curve in the the upper half-plane starting from the boundary by a real-valued function via the Loewner differential equation. This function is called the Loewner driving function of the curve. This theory has played an important role in the development of Stochastic Loewner evolution (by Lawler, Schramm, Werner and others). However, little is understood about the relationship between the driving function and the curve and it is only possible to explicitly calculate the driving function of the curve in a few cases. In this talk, we will consider the Loewner driving function of periodic curves in the upper half-plane and obtain some properties of their driving functions. This is joint work with Carto Wong.

**Siye Wu**, HKU, Hong Kong

*Analytic torsion of twisted de Rham and Dolbeault complexes*

Associated to a topological space there are many invariants. One of them is torsion, which can be defined combinatorially by simplicial method or analytically by zeta-function regularisation when the space is a manifold. In this talk, I will explain how to generalise the analytic torsion by adding a flux form motivated from physics as well as its holomorphic analogue. This is a joint work with Mathai.

**Sai-Kee Yeung**, Purdue University, USA

*Holomorphic one forms, rational and integral points on complex two ball quotients*

We study a problem of Borel on existence of holomorphic one forms on some appropriate unramified covers of a complex two ball quotient, and explain its relation to finiteness of rational and integral point on an arithmetic model of such a manifold, which may be compact or non-compact with finite volume.