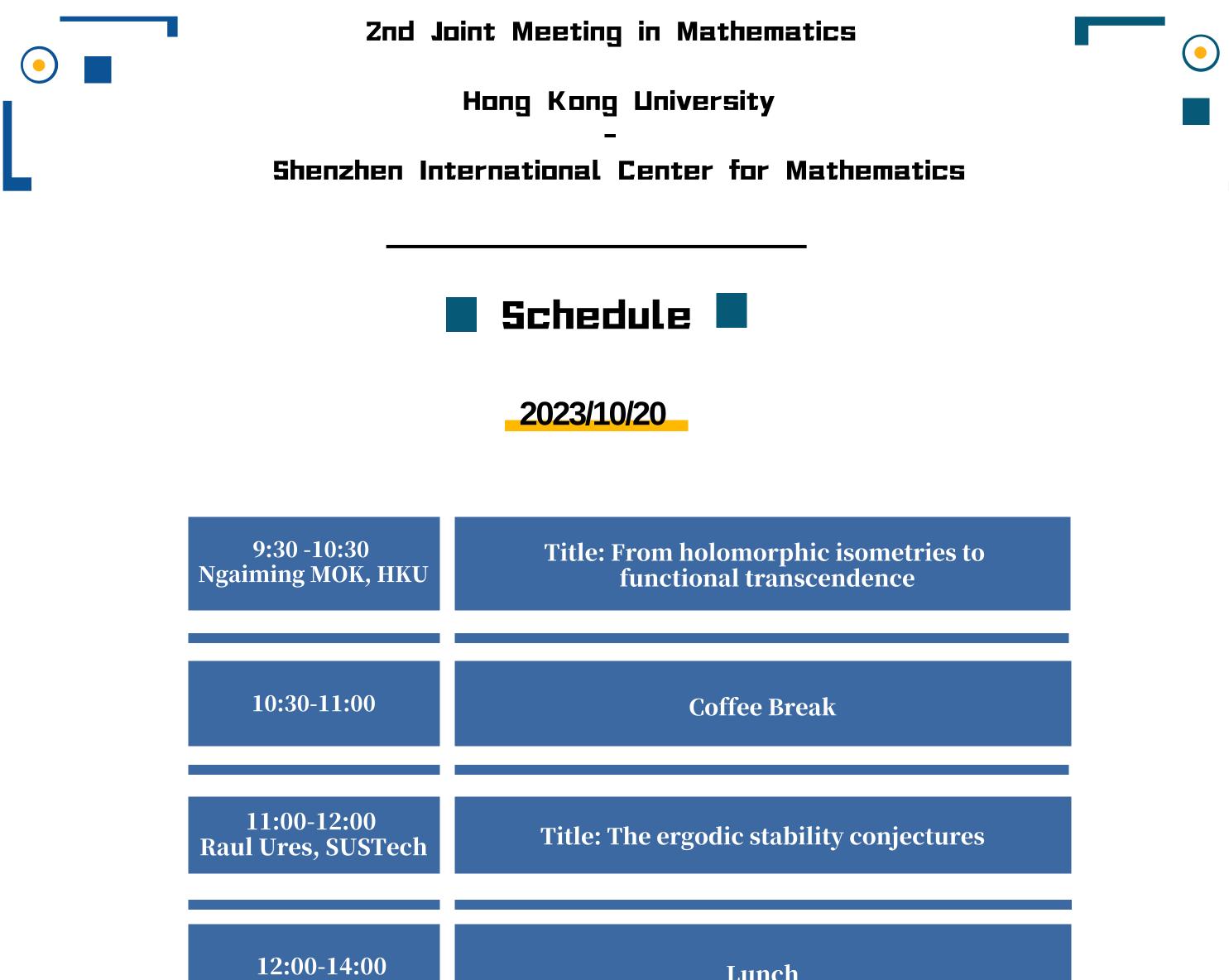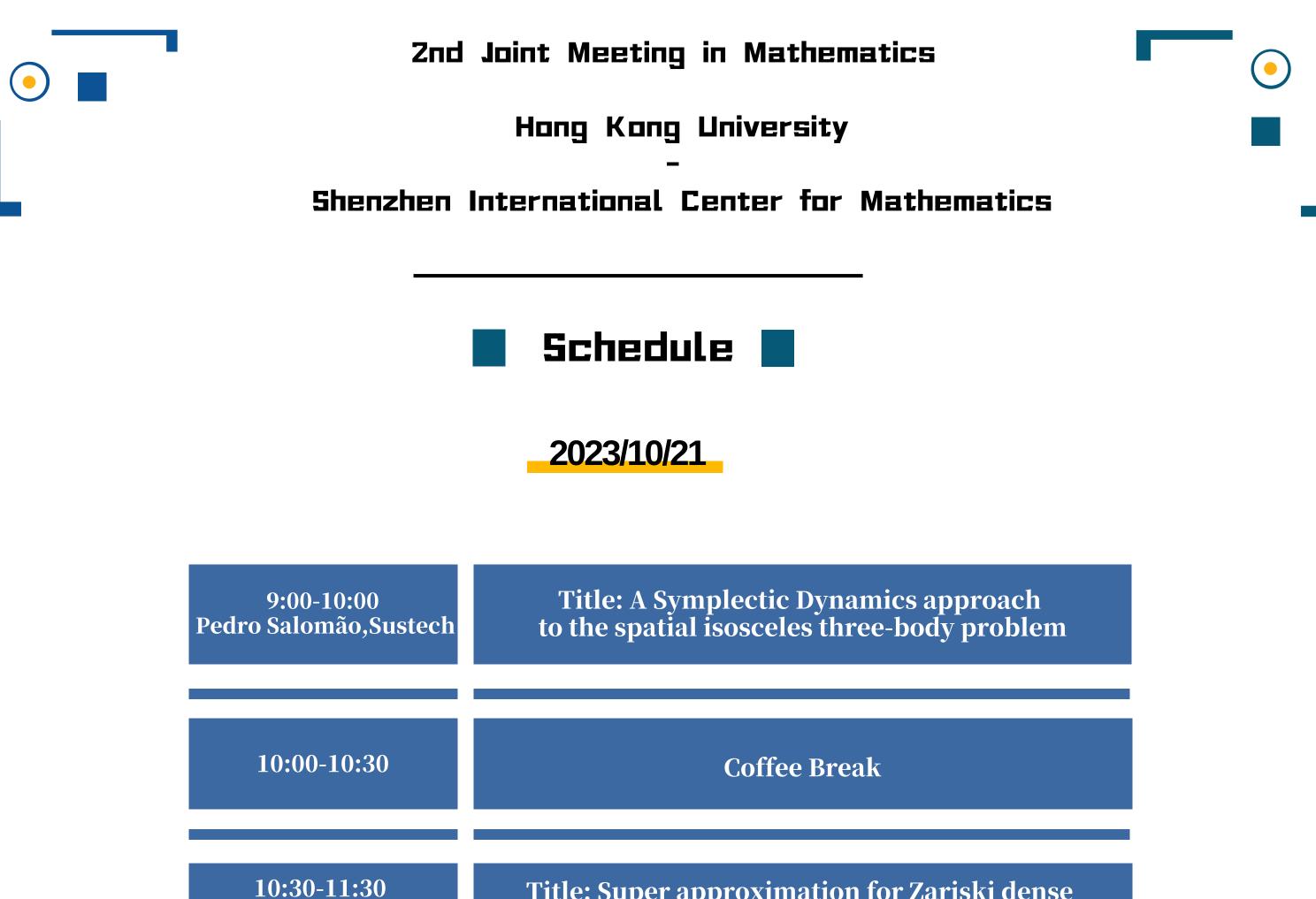# DATE

2023/10/20 - 2023/10/21

2nd Joint Meeting in Mathematics

Hong Kong University
-
Shenzhen International Center for Mathematics

Venue: SICM Lecture Hall
(Room 240A, Taizhou Hall)

2nd Joint Meeting in Mathematics

Hong Kong University
-
Shenzhen International Center for Mathematics

◼ Schedule ◼

2023/10/20

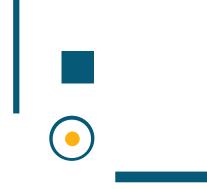|	
9:30 -10:30 Ngaiming MOK, HKU	Title: From holomorphic isometries to functional transcendence
10:30-11:00	Coffee Break
11:00-12:00 Raul Ures, SUSTech	Title: The ergodic stability conjectures
12:00-14:00	Lunch
14:00-15:00 Tuen Wai NG, HKU	Title: An entire function analogue of Smale's 12th problem on centralizersof diffeomorphisms
15:00-15:30	Coffee Break
15:30-16:30 Carlos Gustavo Moreira Sustech	Title:On the Markov and Lagrange spectra
16:30-17:30 Kwok Kin WONG, HKU	Title: Extension of inverses of Γ-equivariant holomorphic embeddings of bounded symmetric domains of rank $\geqslant 2$ and applications to rigidity problems.

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■ Schedule ■

2023/10/21

Time / Speaker	Title
9:00-10:00 Pedro Salomão, Sustech	Title: A Symplectic Dynamics approach to the spatial isosceles three-body problem
10:00-10:30	Coffee Break
10:30-11:30 Xin ZHANG, HKU	Title: Super approximation for Zariski dense subgroups of SL2(Z)×SL2(Z)
11:30-12:30 Yiwei Zhang, Sustech	Title: Typically periodic optimization conjecture for ergodic optimization
12:30 - 13:30	Lunch

From holomorphic isometries to functional transcendence on quotients of bounded symmetric domains

Ngaiming MOK

The University of Hong Kong

Abstract: Consider a K¨ahler manifold (X, g). When g can be expanded in power series, in his seminal work on holomorphic isometries Eugenio Calabi introduced the notion of the diastasis and proved powerful extension theorems on holomorphic isometries from K¨ahler manifolds into space forms such as the projective space equipped with the Fubini-Study metric. On a bounded domain U ⋐ C n we denote by ds2 U the Bergman metric on U (which is K¨ahler). Among bounded domains there are the bounded symmetric domains Ω classified by Elie Cartan such that (Ω, ds2 Ω) are symmetric, Ω = G/K, in the sense of Riemannian geometry. Here Ω ⋐ C n in their standard realizations are bounded domains with semi-algebraic boundaries. In this lecture I will explain: (1) how the study of holomorphic isometries between bounded domains was motivated by problems in arithmetic dynamics, (2) how their solutions were generalized to yield algebraicity results for holomorphic isometries with respect to the Bergman metric, (3) how the study of the asymptotic behavior of holomorphic isometries of the Poincar´e disk led to a uniformization theorem for projective varieties covered by algebraic subsets of Ω, and (4) how the latter serves as a starting point for research in functional transcendence theory concerning XΓ = Ω/Γ, where Γ ⊂ G is an arbitrary lattice. In the special case of arithmetic lattices, (4) has been settled yielding the Ax-Schanuel theorem on Shimura varieties (with extensive generalizations by now) by Mok-Pila-Tsimerman (2019), using techniques involving in particular model theory from mathematical logic, techniques which are no longer available for arbitrary lattices. In the case where X = Ω/Γ is compact and Γ ⊂ Aut(X) is an arbitrary lattice, I will explain how functional transcendence results on X can be proven using analytic techniques starting with the rescaling method on subvarieties of bounded symmetric domains.

The ergodic stability conjectures

Raul Ures

Southern University of Science and Technology

Abstract: Almost thirty years ago C. Pugh and M. Shub proposed their ergodic stability conjecture. This conjecture states that within partially hyperbolic conservative systems there is an open and dense Cr, r>=2, which is ergodic. A system is partially hyperbolic if it has three invariant directions, one contractive, one expansive, and a third with intermediate behavior. Together with F. and J. R. Hertz we proposed a more precise description of this phenomenon for diffeomorphisms in dimension 3. We conjecture that if the topology of the ambient manifold is sufficiently complicated, all conservative partially hyperbolic diffeomorphisms are ergodic. In this talk we will present the state of the art of both conjectures.

An entire function analogue of Smale's 12th problem on centralizers of diffeomorphisms

Tuen Wai NG

The University of Hong Kong

Abstract: Smale's 12th problem asks for a given compact manifold M, whether the subset of all its diffeomorphisms whose centralizers are trivial is dense in Diffr (M). In this talk, we will study an analogous problem for entire functions defined on the complex plane.

On the Markov and Lagrange spectra

Carlos Gustavo Moreira

Southern University of Science and Technology

Abstract: We will present recent results on the classical Markov and Lagrange spectra, which are sets of real numbers which appear naturally in the study of best approximations of real numbers by rational ones, and have extremly interesting geometric and topological structures. In particular, we will discuss some recent results in collaboration with Luke Jeffreys, Davi Lima, Carlos Matheus, Mark Pollicott, Sandoel Vieira and Polina Vytnova on the set difference between these two sets. We will also discuss the relationship of these results with Dynamical Systems and Fractal Geometry.

Extension of inverses of Γ-equivariant holomorphic embeddings of bounded symmetric domains of rank ⩾ 2 and applications to rigidity problems

Kwok Kin WONG

The University of Hong Kong

Abstract: Let X = Ω/Γ be a finite volume quotient of bounded symmetric domain Ω ⋐ C n of rank ⩾ 2. With an aim of studying holomorphic mappings from X into complex Finsler manifolds of negative curvatures, Mok (2004) showed that any nonconstant Γ-equivariant holomorphic map F : Ω → D into a bounded domain D is necessarily an embedding.
In this talk, we show the existence of an extension of F −1 : F(Ω) → Ω to a bounded holomorphic map R : D → C n , called the Extension Theorem. For the proof, we study Fatou's Thorem of admissible boundary values for bounded holomorphic functions on B m and the Cayley projection from Ω onto one of its maximal boundary components. The later notion relies on Mok (2016)'s construction of holomorphic isometric embedding from B m to Ω via the theory of variety of minimal rational tangents. Another ingredient is Moore's ergodicity theory for semisimple Lie groups. The Extension Theorem will be applied to obtain a fibration Theorem and an isomorphism theorem for some holomorphic maps on X.

A Symplectic Dynamics approach to the spatial isosceles three-body problem

Pedro Salomão

Southern University of Science and Technology

Abstract: The spatial three-body problem is the study of the motion of three point masses in $\R^3$ subjected to Newton's universal gravitational law. There exists an invariant subsystem in which two equal masses are symmetric to a fixed axis where the third body moves. As the three bodies always form an isosceles triangle, this subsystem is called the spatial isosceles three-body problem. For certain choices of mass ratio, angular momentum, and energy, the dynamics on the energy surface is equivalent to a Reeb flow on the tight three-sphere. I will introduce some results from the theory of pseudo-holomorphic curves to obtain periodic orbits and global surfaces of section. More precisely, we show the existence of a Hopf link formed by the Euler orbit and a symmetric brake orbit spanning an open book decomposition whose pages are annulus-like global surfaces of section. For large mass ratios, the Hopf link is proved to be non-resonant, forcing the existence of infinitely many periodic orbits. Convexity of the energy surface is also addressed and for each parameter, we identify whether the energy surface is strictly convex, convex, or non-convex. This is joint work with X. Hu, L. Liu and Y. Ou (Shandong University), and G. Yu (Tianjin University).

Super approximation for Zariski dense subgroups of SL2(Z) × SL2(Z)

Xin ZHANG

The University of Hong Kong

Abstract:It is a discovery of Margulis in 1970s that congruence quotients of SL2(Z) can be used to construct expanders, which are certain sparse but highly connected graphs. The Super Approximation Conjecture of Salehi-Golsefidy and Varju gives a precise prediction on which more general subgroups of SLd(Z) have this property. In this talk, I will survey the history of this conjecture, and describe a recent progress by Tang Jincheng and myself that all Zariski dense subgroups of SL2(Z) × SL2(Z) have this property. This progress relies on the development of a key tool in arithmetic combinatorics conjectured by Salehi-Golsefidy.

Typically periodic optimization conjecture for ergodic optimization

Yiwei Zhang

Southern University of Science and Technology

Abstract: Ergodic optimization is the study of problems relating maximizing invariant measures and maximum ergodic averages. In ergodic optimization theory, one important problem is the typically periodic optimization (TPO) conjecture. This conjecture was proposed by Hunt, Ott and Yuan, and Jenkinson, which reveals the principle of least action in the setting of smooth dynamical systems. To be more precise, TPO indicates that when the dynamical system is suitably hyperbolic and the observable is suitably regular, then the maximizing measure is ``genetically' ' supported on a periodic orbit with relatively low period. In this talk, I will report some recent progresses on this conjecture from various aspects.