



Workshop on Complex Analysis

Wednesday, 22 February, 2023

Room 210, Run Run Shaw Building, HKU

[Workshop on Complex Analysis – Institute of Mathematical Research \(hku.hk\)](http://www.imr.hku.hk)

Morning Session

9:30 – 9:35	Ngaiming MOK (Director of IMR, The University of Hong Kong) <i>Opening remarks</i>
9:35 – 10:05	Wei QIAN (City University of Hong Kong) <i>Uniqueness of the welding problem with fractal interfaces</i> [Abstract]
10:05 – 10:35	Henry CHENG (The Hong Kong University of Science and Technology) <i>Liouville's Theorem using D-modules</i> [Abstract]
<i>Coffee / tea break</i>	
11:00 – 11:30	Jonathan TSAI (The University of Hong Kong) <i>Carathéodory balls and proper holomorphic maps on multiply-connected planar domains</i> [Abstract]
11:30 – 12:00	Chengfa WU (Institute for Advanced Study, Shenzhen University) <i>Meromorphic solutions of the autonomous Schwarzian differential equations</i> [Abstract]
<i>Lunch Break</i>	

Afternoon Session

14:00 – 14:30	Yang CHEN (University of Macau) <i>Asymptotic gap probability distributions of the Gaussian Unitary Ensembles and Jacobi Unitary</i> [Abstract]
14:30 – 15:00	Dan DAI (City University of Hong Kong) <i>Asymptotics of Fredholm determinants associated with higher dimensional kernels</i> [Abstract]
<i>Coffee / tea break</i>	
15:30 – 16:00	Chiu Yin TSANG (The University of Hong Kong) <i>Studies of hypergeometric functions using D-modules</i> [Abstract]
16:00 – 16:30	Jiaxing HUANG (College of Mathematics and Statistics, Shenzhen University) <i>Linear difference and differential operators preserving some A-entire functions</i> [Abstract]
16:30 – 17:00	Chung-Chun YANG (The Hong Kong University of Science and Technology) <i>Applications of Nevanlinna's value distribution theory to functional equations and related conjectures</i> [Abstract]
Organizers: Yik Man CHIANG (HKUST) and Tuen Wai NG (HKU)	

Uniqueness of the welding problem with fractal interfaces

by

WEI QIAN

City University of Hong Kong

We give a simple set of geometric conditions on curves $\eta, \tilde{\eta}$ in \mathbb{H} from 0 to ∞ so that if $\varphi: \mathbb{H} \rightarrow \mathbb{H}$ is a homeomorphism which is conformal off η with $\varphi(\eta) = \tilde{\eta}$ then φ is a conformal automorphism of \mathbb{H} . Our result applies to the setting where the interface η is not the boundary of a Hölder domain or even a connected domain. Our motivation comes from the fact that it is possible to apply our result to random conformal welding problems related to the Schramm-Loewner evolution (SLE) and Liouville quantum gravity (LQG). This is based on a joint work with Jason Miller.

Liouville's Theorem using D -modules

by

KAM-HANG HENRY CHENG

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We give basic ideas about using a D -module approach in complex analysis. Specifically, we establish analogues of Liouville's theorem, with the differential operator replaced by various difference operators. The extraction of series coefficients is done algebraically using a residue map that measures the obstruction to having local "antiderivative". The residue map is based on a Weyl algebra or q -Weyl algebra structure satisfied by each corresponding operator. This explains the different senses of "boundedness" required by the respective analogues of Liouville's theorem. This is a joint work with Yik-Man Chiang and Avery Ching.

Carathéodory balls and proper holomorphic maps on multiply-connected planar domains

by

JONATHAN TSAI
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In this talk, we establish the existence of disconnected open balls and the inequivalence of closed balls and the closure of open balls under the Carathéodory metric in some planar domains of finite connectivity greater than 2. This resolves a problem posed by Jarnicki, Pflug and Vigué in 1992. A corresponding result for some higher dimensional pseudoconvex domains is also obtained. Our results follows from an explicit characterization of proper holomorphic maps from a non-degenerate finitely-connected planar domain onto the unit disk which answers a question posed by Schmieder in 2005. This is analogous to Fatou's famous result that proper holomorphic maps of the unit disk onto itself are finite Blaschke products. A parameter space for proper holomorphic maps is also determined which extends a result of Grunsky.

Joint work with Tuen Wai Ng and Chiu Chak Tang

Meromorphic solutions of the autonomous Schwarzian differential equations

by

CHENGFA WU

Institute for Advanced Study, Shenzhen University

It has been proved by Ishizaki that the autonomous Schwarzian differential equations, which admit transcendental meromorphic solutions, have six canonical forms. In this talk, we will present all of these transcendental meromorphic solutions. This talk is based joint works with Liangwen Liao, Jie Zhang and Donghai Zhao.

Asymptotic gap probability distributions of the Gaussian Unitary Ensembles and Jacobi Unitary

by

YANG CHEN
University of Macau

This talk is “really” about the smallest eigenvalue distribution of the Laguerre Unitary Ensembles. In the process one also finds the smallest eigenvalue distribution of Jacobi unitary ensembles. Under something call double scaling, constants in the asymptotic expansions are express interms of the Barnes G-function.

Asymptotics of Fredholm determinants associated with higher dimensional kernels

by

DAN DAI

City University of Hong Kong

In this talk, we consider the Pearcey kernel and the hard edge Pearcey kernel, which appear in random matrix theory and many other stochastic models. They are viewed as higher dimensional kernels in the sense that they can be characterized by 3×3 matrix-valued Riemann-Hilbert problems. We establish integral representations for Fredholm determinants of integral operators with these kernels, which involve Hamiltonians associated with certain nonlinear differential equations. We also derive large gap asymptotics for the determinants and obtain asymptotic statistical properties for the related point processes.

This is a joint work with Shuai-Xia Xu and Lun Zhang.

Studies of hypergeometric functions using D -modules

by

CHIU-YIN TSANG
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In this talk, we develop a D -module approach to solutions to linear differential equations by long divisions of different differential operators. In particular, this explains Weyl algebraic origin of the series solutions of hypergeometric equations, and can also be applied to difference analogue of hypergeometric functions. The approach to q -analogue and/or Bessel functions may also be discussed.

Linear difference and differential operators preserving some \mathcal{A} -entire functions

by

JIAXING HUANG
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We apply Rossi's half-plane version of Borel's Theorem to study the zero distribution of linear combinations of \mathcal{A} -entire functions. This provides a unified way to study linear q -difference, difference and differential operators (with entire coefficients) preserving subsets of \mathcal{A} -entire functions, and hence obtain several analogous results for the Hermite-Poulain Theorem to linear finite (q -)difference operators with polynomial coefficients. The method also produces a result on the existence of infinitely many nonreal zeros of some differential polynomials of functions in certain sub-classes of \mathcal{A} -entire functions.

Joint work with Tuen Wai Ng

Applications of Nevanlinna's value distribution theory to functional equations and related conjectures

by

CHUNG-CHUN YANG

The Hong Kong University of Science and Technology

It's well known that Nevanlinna's value distribution theory is the most useful, powerful and effective tool in dealing with the functional equations of meromorphic functions. In the talk, some unsolved old and new conjectures (mainly posed by the speaker over past decades) that related to admissible meromorphic solutions of functional equations of various types, including differential equations as well as Fermat type of equations, along with the background and progresses of these conjectures will be surveyed or reported, for further studies